Off the Shelf: Concepts of the Calculus

The Concepts of the Calculus, A Critical and Historical Discussion of the Derivative and the Integral, by Carl B. Boyer. New York: Columbia University Press, 1939. Second edition in 1949 titled *The History of the Calculus and Its Conceptual Development.* A Dover paperback of the 1949 edition appeared in 1959 and remains in print today (US$16.95).

Carl Boyer's *The Concepts of the Calculus* (1939) was a substantial contribution to the history of mathematics and a significant signpost in the emergence of the history of science as a professional discipline in America. Note that Boyer was writing about "the calculus" and not "calculus": even at this late stage the subject was not yet regarded as a general part of mathematical knowledge, like algebra or arithmetic,

but a particular mathematical subject area with a definite historical identity and point of origin. The phrasing "a critical and historical discussion" echoed the title of Ernst Mach's classic 1883 *The Science of Mechanics: A Critical and Historical Account of Its Development.* This work in turn was emblematic of a positivist interest in history and the conviction that history could illuminate an understanding of foundations.

In researching his book Boyer drew on the impressive collection of the New York Public Library, a fact that is apparent in the extensive bibliography containing diverse sources in several languages. The work grew out of his PhD research in intellectual history at Columbia University and reflected the seriousness of the professional scholar. In his obituary of Boyer the historian Charles C. Gillispie (*Isis* 76 (1976): 610–614) comments on Boyer's determination, as a mathematics instructor in the 1930s, to carry out research in the history of the subject (p. 611): "What resolution it required to persevere in a discipline in which all the premium was on the creation of new pieces of mathematics—in which old mathematics was often denatured or patronized as childish—can begin to be appreciated only since sociological study has brought home the force and nature of scientific norms of behavior."

In his preface Boyer stated that his goal was to provide a "critical account of the filiation of the fundamental ideas of the subject." His book was a contribution to a genre of historical writing that was popular in the first half of the twentieth century but is less so today: a survey work focused on a specific concept or a few concepts. Other representatives of this genre were Duane Roller's *The Early Development of the Concepts of Temperature and Heat* (1950), Georges Canguilhem's *La formation du concept de réflexe aux XVIIe et XVIIIe siècles* (1955), and Max Jammer's *Concepts of Space: The History of Theories of Space in Physics* (1954). In 1940 Arthur O. Lovejoy founded the *Journal of the History of Ideas*, providing a formal publication venue for the atomistic conception of intellectual history that was current at the time.

Canguilhem was the most explicit in articulating a philosophical basis for looking at concepts. Cristina Chimisso in her book *Writing the History of the Mind: Philosophy and Science in France, 1900 to 1960s* (2008), observes (p. 158):

Canguilhem's defence of a history of concepts partially independent of theories and indeed metaphysical assumptions introduces a new historiographical perspective. ...He not only believed that changes of worldviews took place in a slow and fragmentary manner, but also and crucially that concepts could survive within different worldviews, metaphysical assumptions and indeed theories. ...For him the scientificity of a concept, or its potential scientific value, does not appear to depend on his general assessment of theories, let alone worldviews and mentalities, in which it emerges.

The appeal of writing a history of a concept may also have been motivated by simple reasons of narrative. The concept stands as a proxy for the larger subject or theory. Accounts of the development of a concept are analogous to road-trip narratives in fiction, recounting the adventures of the hero on the road. The concept is the hero, the historical line of development is the road, and the modern concept is the hero at the destination. (Just two years earlier Oxford fellow J. R. R. Tolkien published *The Hobbit* with its tale of the eventful journey of Bilbo Baggins to the Lonely Mountain.)

The focus on the development of concepts through time may reflect as well an embrace of the metaphor of a plant or animal organism. The concept undergoes a progressive development, moving in a directed and pre-determined way from its origins to an adult and completed form. It is possible to identify modern characteristics of the subject in its earlier history, just as it is possible to identify incipient adult characteristics of an organism in its early formation and development. The possibility of introducing anachronisms is almost inevitable in such an approach, and to a certain degree this is true of Boyer's book. The chapter on the period from 1580 to 1680 is titled "A Century of Anticipation." The chapter on the eighteenth century is titled "The Period of Indecision." The nineteenth century is the quest finally realized: "The Rigorous Formulation."

Boyer's book is a modern classic, and remains today a stimulating and highly informative study of the history of analysis. Nevertheless, a large body of work on this subject has appeared since its publication, and parts of it need to be revised and supplemented. We will consider here only the chapter on the eigh-

teenth century. Boyer correctly identified the significant role played by Leonhard Euler and Joseph-Louis Lagrange in moving away from a conception of calculus as "fine geometry" to a mathematical subject in its own right. He wrote: "Most of his [Euler's] predecessors had considered the differential calculus as bound up with geometry, but Euler made the subject a formal theory of functions which had no need to revert to diagrams or geometrical conceptions." (p. 243 of the Dover edition). Nevertheless, he seemed to view the eighteenth-century work as exploratory or approximative as the subject moved inexorably in the direction of the arithmetical limit-based approach of Augustin-Louis Cauchy and Karl Weierstrass. Over the past several decades historians have documented in some detail the distinctive conception underpinning the work of the eighteenth-century formalists and the philosophical vision expressed therein. Another noteworthy development was the invention of non-Archimedean versions of analysis, beginning in the 1950s. This last development showed that logically and psychologically it was not necessary to perceive earlier work with infinitesimals as simply a naïve precursor to a modern rigorous formulation.

Craig Fraser