Book Reviews brings interesting mathematical sciences and education publications drawn from across the entire spectrum of mathematics to the attention of the CMS readership. Comments, suggestions, and submissions are welcome.

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Les comptes-rendus de livres présentent aux lecteurs de la SMC des ouvrages intéressants sur les mathématiques et l'enseignement des mathématiques dans un large éventail de domaines et sous-domaines. Vos commentaires, suggestions et propositions sont le bienvenue.

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Leonhard Euler: Mathematical Genius in the Enlightenment

by Ronald S. Calinger

Princeton University Press, 2015

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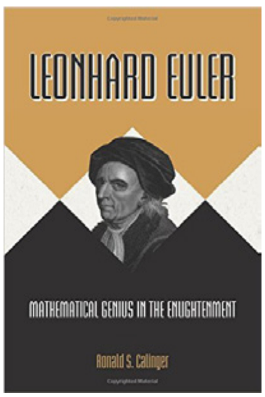
Reviewed by *Craig Fraser* (University of Toronto)



Ronald Calinger's biography provides a year-by-year account of Euler's life and mathematical work, from his youth in Basle through his long career in St. Petersburg and Berlin. Euler's research projects, academic relations and collegial connections are chronicled in detail, and there are accounts of his many contributions to mathematics. The book is an impressive and very substantial contribution to Euler studies.

Writings on Euler over the past fifty years are divided into two groups, with mathematicians focusing on the heritage of Euler's mathematics and its interest to a modern reader, while historians have examined the emergence and development of Euler's theories in the historical context of eighteenth-century exact science. Calinger draws on both bodies of work as well as the existing biographical studies (including some articles of his own) to construct this portrait of Euler. The amount of information and detail here about the administrative and institutional context of Euler's life is unsurpassed in biographical writings in the history of mathematics.

Various extra-mathematical subjects are broached in the course of Calinger's book, including Euler's views on theology, his philosophical disagreement with followers of Christian Wolff on the subject of monodology, his interest in teaching, his engagement with the game of chess, his administrative oversight of public works projects, the constant academic politics, and his relations with his political masters. Also covered are family matters, his two marriages and the arrival of his thirteen children (all by his first wife), eight of whom died in infancy or early childhood and only three of whom – all sons – outlived him.

Euler's career in St. Petersburg and Berlin was carried out against a backdrop of war and social upheaval. The War of the Austrian Succession, the Silesian Wars and the Seven Years' War occupied the attentions of the leaders of Russia and Prussia from the early 1740s to the late 1750s. Some of the details Calinger reports on provide a glimpse of how the larger events of the time affected Euler and his family. National treasuries could not afford barracks for all of the soldiers of their large standing armies (containing not a few "social misfits, criminals and vagrants") and private homes were required to billet these men. At one point during his first years in St. Petersburg eight soldiers were billeted at Euler's residence in a shed at the rear of his house, and they "could be crude, rowdy and dishonest, and their heavy smoking discolored the building's interior." When Euler moved to Berlin he negotiated with Frederick II to ensure that he would not be required to billet soldiers at his home. The journey from St. Petersburg to Berlin included a three-week rough voyage on the Baltic Sea by Euler and his young family, during which only Euler was free of seasickness.

In addition to his extensive contributions to analysis and number theory, Euler carried out research on practical projects, from the theory of music, the use of magnetic declination lines to determine longitude, the science of ballistics (of utmost interest to his military sponsors), to the stability of ships and the optics of telescope construction. He also contributed to theoretical mechanics and the rotation of rigid bodies. (Calinger uses the term "rational mechanics," a designation that does not appear in Euler's writings and came into substantial use in English only in the second half of the twentieth century.) Euler authored important writings on the mathematical motions of fluids and the theory of engineering structures, and contributed extensively to physical astronomy (what later became known as celestial mechanics).

Enormously creative in his own right, Euler was quick to recognize important new mathematical advances by others, sometimes conveyed to him in personal letters, sometimes presented in published articles. He would clearly credit the discoverers and go on to make very detailed and fundamental contributions to the subject. He did that for Jean d'Alembert's work on the derivation and integration of partial differential equations, Alexis Clairaut's solution to the lunar three-body problem, Giulio Fagnano's researches on elliptic integrals, and Joseph Lagrange's delta algorithm in the calculus of variations. Inevitably tensions concerning priority and approach arose in his relations with other researchers, but Euler displayed pragmatism and good sense in navigating these shoals.

Despite failing eyesight and other health problems, Euler remained remarkably productive into his seventies. Johann Bernoulli III reported in 1778 that "he cannot recognize people by their faces, nor read black on white, nor write with pen and paper; yet with chalk he writes his mathematical calculations on a blackboard very clearly and in rather normal size; these are immediately copied by one of his assistants ..., and from these materials are later composed memoirs under his direction." (Quoted in Emil A. Fellmann's Euler (Basle: Birkhäuser), p. 119, reproduced on p. 519 of Calinger's book.)

Although Calinger does not identify any overall pattern in the evolution of Euler's mathematical approach during his sixty years of active work, it appears that in his later researches there was a stronger synthetic conception of the mathematical material. In some of his last investigations he began to draw the outlines of a theory of functions of a complex variable. In researches completed up to his death in 1783 he obtained results in this area that would be the starting point for investigations of Simon Laplace, Siméon Poisson and Augustin Cauchy, a line of research that led some sixty years after Euler's death to a new and major branch of analysis.

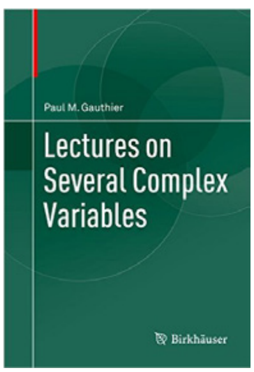
Lectures on several complex variables

by Paul M. Gauthier

Birkhäuser, 2014

ISBN: 978-3-319-11510-8

Reviewed by *Thomas Bloom* (University of Toronto)



The origins of the theory several complex variables go back to the beginning of the twentieth century, with the discovery by F. Hartogs, E. E. Levi, H. Poincaré, (and others) of phenomena in several complex variables which had no parallel in the one variable case. Central problems in the theory were solved by the Japanese mathematician Kiyoshi Oka in the 1930s and 1940s. His theory was refined and formulated using the theory of sheaves by H. Cartan and J. P. Serre in seminars in Paris in the early 1950s. More explicit solutions to results in the general theory were given in the 1950s by H. Grauert and L. Hörmander. By the 1960s the basic theory and different approaches to it were known.

Textbook/reference books presenting the general theory or some aspect of it soon followed and today there are many such books. Three excellent ones which present the general theory are the classic book of Hörmander using partial differential equation methods and giving highly useful estimates to basic existence theorems [3], the three volume set of Gunning with a detailed presentation of the theory of sheaves [2], and the online book of Demailly where the emphasis is to present the theory on complex manifolds [1]. The book of Hörmander, at about 250 pages, is very tersely written and the books of Gunning and Demailly each run about 500 pages. It normally takes a dedicated and qualified graduate student a full year of concentrated study to master the essence of any one of these books.

The book under review, at a mere 100 pages or so, does not attempt to present the theory in general nor does it give a systematic or complete presentation of the topics discussed. Rather, it aims to give an introduction to the subject, but nevertheless to present complete proofs of some results in several complex variables.

The only prerequisites are standard undergraduate mathematics.

The first phenomena discussed is the example of Hartogs:

$$H = \{z \in \mathbb{C}^2 : |z_1| < 1/2, |z_2| < 1\}$$
$$\cup \{z \in \mathbb{C}^2 : |z_1| < 1, 1/2 < |z_2| < 1\}.$$

Every analytic function on H extends to a holomorphic function on the unit polydisc

$$D = \{z : |z_1| < 1, |z_2| < 1\}.$$

In the terminology of several complex variables H is not a *domain of holomorphy*. In keeping with the spirit of the book, for the complete characterization of domains of holomorphy and of course for the proofs, the reader is referred to other texts.

Plurisubharmonic functions, the Monge-Ampère equation and the related Dirichlet problem and the characterization of domains of holomorphy via pseudoconvexity are, however, discussed.

The Weierstrass preparation theorem, which is the basis of the study of the local geometry of the zero sets of holomorphic functions, is proved.

It is also proven that the natural domain of definition of an analytic function (i.e. the maximal domain into which it can be analytically continued) is a complex manifold spread (i.e. with a natural local isomorphism) over $\mathbb{C}^n$. This follows an extensive section on complex manifolds including a careful presentation of specific higher dimensional complex manifolds (projective space, Grassmanians, tori, quotient spaces, etc...) and also Hermitian, symplectic and almost complex manifolds.

The book concludes with a discussion of the difficulties in defining mereomorphic functions in several variables. The Cousin problem of writing a mereomorphic function globally as a quotient is briefly mentioned, as is the definition of subvariety.

The target audience for the book is graduate students, more ambitious undergraduate students or research mathematicians whose specialty is not several complex variables. The author engages the reader with well-chosen, interesting problems which advance the theory. The book is written with a delightful light touch and provides a very accessible introduction to several complex variables.

References

[1] J. P. Demailly, **https://www-fourier.ujf-grenoble.fr/~demailly/manuscripts/agbook.pdf**

[2] R. C. Gunning, Introduction to Holomorphic Functions of Several Variables, three volumes, Wadsworth and Brooks/Cole (1990).

[3] L. Hörmander, An Introduction to complex analysis in Several variables, 3rd edition, North Holland (1990).