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★**History of virtual work laws.**

A history of mechanics prospective.

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Early versions of the principle of virtual displacements appeared in treatises on mechanics going back to a pseudo-Aristotelian tract on mechanics in antiquity. The idea behind the principle was well known in the seventeenth century and used to analyze the equilibrium conditions of machines. Early in the eighteenth century researchers formulated what was called the principle of virtual velocities as a general condition for static equilibrium. Joseph Lagrange's *Analytical mechanics* of 1787 was based on a generalized form of the principle that applied to both statics and dynamics. In Lagrange's conception, the principle was well suited to the analytical development of mechanics using calculus techniques. In this approach scalar quantities representing what later would be called energy, potential, and work occupied a fundamental place in the theory and provided an alternative to a formulation in terms of force, understood as a proto-vector quantity. At the end of the eighteenth century and well into the nineteenth century the principle of virtual displacements was an object of lively discussion and investigation.

The book under review is a survey work that covers the whole span of history from antiquity to the middle of the nineteenth century, supplemented by a short chapter on Pierre Duhem's contributions to thermodynamics around 1900. Written from the perspective of an engineer, it is a substantial contribution to the history of physical science and will become a standard reference source for the history of mechanics. The book is divided into two parts. The first is an examination of developments in the "science of weights", beginning in antiquity and the Middle Ages, and ending with various results achieved by early modern investigators. The second part commences in the eighteenth century, with the formulation of general principles that became the basis of comprehensive approaches to mechanics. The author gives a detailed account of the attempts by researchers around 1800 to prove the principle of virtual velocities. Included here are the demonstrations of Lagrange, Lazare Carnot, Gaspard de Prony, André Marie Ampère, Joseph Fourier, Pierre Laplace, Louis Poinsot, Siméon Poisson, Jean Duhamel and Gustave Coriolis. A chapter is devoted to the relatively unknown Italian group of researchers Vittorio Fossombroni, Girolamo Saldani, and Gabrio Piola, who in the early nineteenth century debated the validity of the principle. The book concludes with an examination of how the virtual displacement idea was extended to hydrodynamics and elasticity up to the 1860s. It is worth noting that the name "virtual work" appears nowhere in any of the writings examined in this book, being terminology that only became standard in the twentieth century.

Since physical laws are inductive generalizations, one might well ask what it means to prove the principle of virtual displacements. The extent to which the principle has nontrivial physical content derives from the way it applies to a constrained system of bodies, where the constraints are given in some mathematically tractable form and the forces in the system consist only of external forces. It is necessary to show that for a reasonably broad class of such constrained systems, the law is valid. Following a reductionist approach, which was the most common approach taken, one begins with some form of what are usually called Newton's laws and combines these with assumptions about the mathematical constraints to show that the principle holds. By contrast, in a foundationalist approach, followed by such authors as François Servois, one begins with a postulate that expresses the truth of the principle in some primitive setting, this truth presumably being inferred from experience, and then proceeds to show that the principle may be extended to a range of constrained systems.

One facet of the subject not explored by the author concerns theorizing that involved maximization or minimization of quantities. The formulation of virtual displacement principles in the eighteenth century was associated with action laws in dynamics, and the calculus of variations provided the requisite mathematical tools. In the last chapter minimum principles of Carl Gauss and Duhem are mentioned, but the variational aspect of the subject receives little discussion. It is of some interest that it is possible to write a lengthy history of virtual work laws with very little reference to ideas of optimization. Certainly such ideas were and are of scientific and philosophical interest and have since 1700 been closely connected to the formal mathematical development of mechanics.

Throughout the book there are quotations and passages from primary sources, translated into English. In appendices these excerpts are given in the original languages in which they were written. The bibliography contains extensive references to the secondary literature, but the use of this literature is not documented within the main text. There is an index of persons but no subject index. *Craig G. Fraser*