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Cauchy, Augustin-Louis

★**Cauchy's *Calcul infinitésimal.***

A complete English translation.

Translated from the 1823 French original and with a preface by Dennis M. Cates.

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Until recent decades the major French mathematical classics of the eighteenth and nineteenth centuries remained untranslated into English. At the time of their original appearance, French was the primary international language and these works would have been accessible to most English-speaking mathematicians. This was also true as time passed and they became primarily the subject of historical interest. However, in today's world such familiarity can no longer be assumed and there is a real need for good translations. The book under review is the first English translation of Augustin-Louis Cauchy's *Resumé des leçons sur le calcul infinitésimal*, originally published in 1823. The *Resumé* as well as two other books Cauchy wrote in the 1820s were based on lectures that he delivered at the École Polytechnique, the famous military engineering school founded in Paris during the French Revolution. In these writings Cauchy put the calculus on an arithmetic basis involving the concepts of function, limit, continuity and summation. Older geometric conceptions as well as Joseph-Louis Lagrange's algebraic mathematical philosophy were rejected in favour of a more rigorous foundation in which the numerical continuum became the fundamental theoretical construct.

Historians of natural science have called attention to obstacles in the way of any easy understanding or appreciation of past science. Noel Swerdlow, a historian of ancient and early modern mathematical astronomy, has observed that: "Unlike literature and the arts, which possess a kind of immortality and can for the most part speak to anyone who takes the trouble to examine them with care and sensitivity, earlier science is mostly dead and mostly technical, requiring various kinds of prior knowledge just to be intelligible" [Amer. Scholar **48** (1979), no. 4, 523–531 (p. 523)]. Hence professional translations and critical editions of original scientific works are accompanied by extensive commentary, footnoting and bibliography.

Swerdlow's perspective is not shared by all mathematicians looking at the history of their subject. There is often only a limited awareness that a past mathematical subject is embedded in a conceptual paradigm different from today's theory. The translation under review has only a short introduction and eschews all but the most cursory footnotes. The translation is accurate if somewhat literal and awkward at times. (For example, "je n'ignore pas que l'illustre auteur" is rendered as "I do not ignore that the illustrious author", "la plupart des géomètres s'accordent maintenant à reconnaître" as "the majority of mathematicians are now in accordance to recognize", and so on.) Readers interested in consulting the original French text may easily do so through the website Gallica.

The translator's intent is to hew close to the original so that the reader may enjoy (p. viii) "as honest and true Cauchy experience as possible" and "can experience Cauchy's work as it may have been like 200 years ago". The reader is also provided at the end with a selection of references to historical writing over the past fifty years on Cauchy's analysis. Included here are standard works by such historians as Bruno Belhoste, Umberto Bottazzini, Judith Grabiner and Ivor Grattan-Guinness, among others.

We can only hope that the hypothetical reader envisaged by the translator fares better than did those of Cauchy's own time. His new doctrine was received poorly by both students and academic administrators of the day. The students staged walkouts from his lectures (a serious matter at a military school), and the head of the École complained to the Ministry of War that Cauchy's lectures involved a "luxury of analysis no doubt appropriate for papers to be read at the Institute but superabundant for the teaching of the Students at this School" (quoted in [I. Grattan-Guinness, Amer. Math. Monthly **112** (2005), no. 3, 233–250 (p. 242); MR2125385]).

Although Cauchy's *Resumé* can be rightly called the first text on what would become real analysis, it also dealt with topics involving functions of a complex variable. In the eighteenth-century algebraic approach there was no logical difference between real and complex analysis. Functions were composed of analytic expressions involving variables, constants, algebraic and transcendental operations, and even such entities as $\sqrt{-1}$. The familiar modern concepts of domain and range were not part of the theory. This way of thinking was still influential in the development of Cauchy's research. It should be remembered that the fundamental concept of modern complex analysis, the concept of the complex plane, was only formulated in the 1830s. One of the fascinating features of Cauchy's pioneering researches of the 1820s was how he was able to develop some of the major parts of complex analysis within a theoretical framework that was incomplete and very much in a state of flux.

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