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8.1

Classical mechanics

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1 INTRODUCTION

Classical mechanics may be defined most generally as the study of the equilibrium and motion of bodies based on the principle of inertia, and employing the mathematics of the differential and integral calculus. It refers to the range of theories in dynamics and material science that have developed historically from the seventeenth century to the present. It emphasizes formal abstraction and mathematically grounded concepts as tools in the investigation of physical phenomena, but also recognizes the essential role of experiment and engineering experience in achieving successful theories. The adjective 'classical' is used to distinguish the subject from two more recent developments: Albert Einstein's special theory of relativity of 1905, and quantum mechanics, invented by E. Schrödinger and W. Heisenberg in the 1920s (§9.13 and §9.15).

In the seventeenth century, several traditions of quantitative research emerged involving distinct methodological and physical precepts. The most prominent developments were connected with advances in astronomy, with the new theories of Nicolaus Copernicus and Johannes Kepler. The beginning of classical mechanics, conventionally taken to be Isaac Newton's *Philosophiae naturalis principia mathematica* (1687), provided an appropriate physics for a heliocentric astronomy. The celebrated 'Newtonian synthesis' was the brilliant culmination of the pioneering work of Galileo Galilei and others in inertial mechanics. Celestial dynamics and problems of stability continued to be a central area of investigation in the history of the subject.

Astronomy was by no means the only source of problems that led to classical mechanics. Historical writing over the past few decades has discredited the view, originating in the late nineteenth century, according to which the basic theoretical structure of the subject was supposed to have emerged in its entirety with the publication of the *Principia*. This work successfully treated only a limited range of phenomena – the central-force dynamics of freely moving particles – and employed mathematical methods that have since been discarded. Theoretical engineering and technology also contributed problems, concepts and techniques to major branches of the subject. More generally, although specific physical hypotheses put forward by René Descartes and Christiaan Huygens were rejected in the eighteenth century, Cartesianism as a scientific philosophy continued to exert a profound influence on mechanical thought.

2 NEWTON'S *PRINCIPIA*

The *Principia* was written by Newton from 1684 to 1687 in a period of intense intellectual activity that is probably unmatched in the history of science. At the time he was Lucasian Professor of Mathematics at Cambridge, and in his early forties. He presented the treatise in the neo-Euclidean style favoured by mathematical scientists of the early modern period. It begins with a preliminary set of definitions and 'axioms, or laws of nature', and is followed by three parts, or 'books'. Book I, which opens with eleven mathematical lemmas containing Newton's version of the calculus, a kind of geometrical theory of limits, is a systematic treatise on particle dynamics; Book II, the least successful of the three, investigates the motion of bodies in resisting media; and Book III introduces the universal law of gravitation and applies the mathematical theory of Book I to the Solar System.

The *Principia* grew from a draft essay on dynamics entitled 'De motu', written in 1684. This essay, which would be incorporated into the opening sections of Book I, contains Newton's significant theoretical innovations in dynamics. An indication of his approach is provided by the results published as Propositions 1, 6 and 11 of Book I. He considers a particle P acted upon by a force directed toward a fixed centre S (Figure 1). In Proposition 1, Newton uses geometrical–infinitesimal techniques to show that, as a consequence of the action of the force and the inertial motion of P, the line connecting S and P sweeps out equal areas in equal times. In Proposition 6 he introduces a measure for the force, and in the corollaries uses the area law to express this measure entirely in terms of spatial quantities. With reference to Figure 1, he shows that the force acting on P is proportional to $(QR)/(SP)^2(QT)^2$. This result prepares the way for the initial problem of his dynamics, to calculate the force from a knowledge of the orbit or trajectory of P. The inspiration for the entire treatise developed from Newton's discovery that this purely technical–mathematical question led to a coherent and substantial body of results. Thus, in Proposition 11 he shows mathematically that if the trajectory is an ellipse with S at one focus, then the force is inversely proportional to the square of SP. In

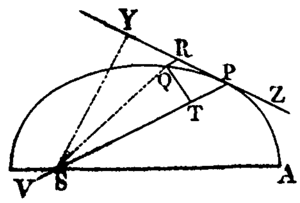

Figure 1 The equal-areas law, from Newton's *Principia* (1687)

subsequent propositions he extends this result to cases in which the trajectory is a parabola and hyperbola.

Although Proposition 11 was presented by Newton as a mathematical theorem, it had an immediate application to planetary motion. The planets were known by Kepler's laws to move in ellipses with the Sun at one focus. If one assumed they moved as a consequence of an attractive force directed towards the Sun, then it was clear from Proposition 11 that the force would vary inversely as the square of the distance from the Sun. Proposition 45 provided further confirmation of this law. For a particle moving about a centre in a closed trajectory, the line of apsides is defined as the axis joining the closest and most distant points of approach. Consider a force law of the form k/r^n. Newton showed mathematically that if n is any number other than 2, then the line of apsides will experience a regular rotation. Since such a rotation was not observed in planetary motion, it followed once again that the planets were governed by an inverse-square law.

The *Principia* was a large, difficult work containing many completed results and many suggestions for future research. It became the cornerstone of classical inertial physics and provided the paradigm for subsequent research in physical astronomy. Newton was fortunate in having chosen to investigate a range of physical phenomena – the dynamics of particles – that was so entirely amenable to systematic analysis in terms of the physical concepts and mathematical techniques of the period. The scientific significance of his theory was potentially far-reaching. If one assumed that all phenomena were derived from the interaction by forces among corpuscles or atoms, then it followed that Newtonian particle dynamics was in principle the ultimate foundation for all of physical science.

3 ANALYTICAL MECHANICS

Newton had invented an analytical calculus in his study of Cartesian algebraic geometry in the 1660s. In the next two decades he developed a strong aversion to all aspects of Descartes's philosophical and scientific thought. In the *Principia* he avoided analytical techniques, preferring instead a thoroughly geometrical formulation of dynamical theory. The archaic appearance of his treatise to a modern reader is due in no small part to the rather peculiar mathematical idiom he adopted.

The eighteenth-century progress of theoretical mechanics occurred almost entirely on the Continent, using the mathematics of the Leibnizian calculus (§3.2). The *Principia* became historically influential once its theory had been absorbed by researchers working in the scientific academies of Paris, Berlin and St Petersburg. The French priest Pierre Varignon wrote a series of memoirs beginning in 1700 on orbital dynamics that employed Leibnizian analytical methods. He gave an elegant demonstration of *Principia*, Book I, Proposition 11 that replaced Newton's involved geometric reasoning by a simple algorithmic procedure. Varignon's contemporaries John I Bernoulli and Jakob Hermann also contributed to his subject, providing a uniform mathematical treatment of the so-called 'inverse problem' in which it was required to determine the orbit from a knowledge of the force law.

The analytical development of dynamics is illustrated in the differential-equation form of what is known today as Newton's second law:

$$M\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = F_x, \qquad M\frac{\mathrm{d}^2y}{\mathrm{d}t^2} = F_y, \qquad M\frac{\mathrm{d}^2z}{\mathrm{d}t^2} = F_z, \tag{1}$$

where M is the mass of the body, x, y and z are its spatial coordinates, and F_x, F_y and F_z are the components of the total force acting on it. Newton's original statement of the law was verbal and referred to increments of velocity rather than to acceleration. Only slowly, in the first half of the eighteenth century, did equation (1) come to be recognized as a fundamental principle. Special cases of the law had appeared in the orbital dynamics of Varignon and John I Bernoulli, where it had constituted the mathematical foundation of the investigation. In more complicated mechanical systems researchers tended to analyse the phenomena from first principles in ways that obscured the possibility of recognizing (1) as the operative physical law. Part of the difficulty was that sophistication in using the calculus developed gradually, and progress in the formation of new physical concepts were even slower. In his *Traité de dynamique* (1743), the young French scientist Jean d'Alembert employed a relation connecting force and time derivatives to investigate the motion of a heavy hanging chain. Leonhard Euler, perhaps the greatest theorist of eighteenth-century exact science, introduced equation (1) as an explicit general law in his 1750 memoir 'Découverte d'un nouveau principe de mécanique'; the title itself suggests the novelty and originality of these equations, introduced over sixty years after the appearance of Newton's *Principia*. In this paper he used it to find the 'Euler equations' for the rotation of a rigid continuous body.

Mathematical mechanics developed more broadly in the eighteenth and nineteenth centuries in association with advances in the theory of infinite series, the calculus of variations, the theory of ordinary and partial differentiation, and differential geometry. In his investigation of the vibrating string in 1747, d'Alembert derived a second-order partial differential equation, the wave equation, and integrated it in terms of arbitrary functions (§3.15). His research started an interesting debate within mathematics on the foundations of analysis and the function concept. Conversely, results in the theory of partial differentiation contributed to an understanding of the behaviour of fluids, elastic beams and continuous media in general. As the eighteenth century progressed, links between the two subjects became deeper and more diverse.

4 THE ESTABLISHMENT OF VARIATIONAL MECHANICS

The establishment of variational mechanics was largely the work of Euler and Joseph Louis Lagrange. Although Lagrange's *Méchanique analitique* (1788) is usually cited as the definitive presentation of the subject, the theory was developed earlier, by Euler between 1740 and 1750 and by Lagrange between 1760 and 1780. Euler provided some of the essential ideas, while the systematic mathematical elaboration of the theory was Lagrange's achievement.

Variational mechanics had its origins in the rule for equilibrium known as the principle of virtual velocities. This principle was a basic axiom in the medieval statics of Jordanus de Nemore, and was discussed by both Descartes and Galileo in the seventeenth century (§2.6). The formulation of the principle common in the eighteenth century appeared in a letter of 1717 from John I Bernoulli to Varignon. Consider a constrained system of bodies in equilibrium with respect to a set of applied forces, and suppose that the system is subjected to a small disturbance which imparts to each mass m a 'virtual' velocity w. This velocity must necessarily be compatible with the constrains in the system. The principle asserts that the sum of the product of the forces and the virtual velocities will be zero:

$$\sum Fw\cos\theta = 0, \tag{2}$$

where F is the applied force acting on m, and θ is the angle between the directions of F and w.

The principle of live force was also important in the establishment of variational mechanics. This principle stipulated that the motion of a dynamical system satisfies the equation

$$\sum mv^2 + 2\Phi(x, y, z, \ldots) = \text{constant}, \tag{3}$$

where v is the speed of a typical mass m, and the variables x, y and z denote its position. The quantity $\Sigma\, mv^2$ was known as the 'live force' of the system, and the expression $\Phi(x, y, z, \ldots)$ was obtained by integrating the forces over the spatial variables. In later mechanics, Φ would be called the potential function of the system.

Equation (3) was derived for particle dynamics, elastic collision and the motion of constrained bodies. It was obtained for a particle acted upon by a central force in Propositions 39–41 in Book I of Newton's *Principia*, and was a standard relation in the orbital dynamics of Varignon and John I Bernoulli. For constrained systems the principle took on a special significance. The forces of constraint do not contribute to the quantity Φ in equation (3); only applied forces need to be considered in calculating this integral. Assume now that, in a given configuration of the system, the velocity of each body is zero. If it is further supposed that in this configuration the quantity Φ is a minimum, then it follows (because $\Sigma\, mv^2$ is always positive) that the system is in static equilibrium with respect to the given applied forces.

It was evident that in particular examples the condition that Φ be a minimum yields precisely the same relation for equilibrium as does the principle of virtual velocities. Since the latter was a fundamental principle, it seemed that the science of statics could be derived from a single variational law asserting that in equilibrium a determined quantity – the integral Φ – is a minimum.

Euler introduced the term 'effort' to denote Φ and called the statement that Φ be a minimum in static equilibrium his 'law of rest'. Following his contemporary Pierre de Maupertuis, he generalized the law of rest to dynamics, thereby obtaining the celebrated principle of least action. This principle asserts that among all curves joining two points in a plane, a particle follows the curve for which the integral $\int mv\,\mathrm{d}s$ is a minimum. Euler first introduced the principle in an appendix to his treatise of 1744 on the calculus of variations. He considered a particle of unit mass with Cartesian coordinates x and y moving freely under the action of a central force. Using the equation of live force, $\frac{1}{2}v^2 + \Phi(x, y) = c$, he wrote the action integral $\int v\,\mathrm{d}s$ as

$$\int [2(c-\Phi)]^{1/2}(1 + y'^2)^{1/2}\,\mathrm{d}x, \quad y' := \mathrm{d}y/\mathrm{d}x, \tag{4}$$

a definite integral that is to be evaluated between the initial and final values of the variable x. By means of mathematical variational methods, he showed that the problem of extremalizing this integral leads to the same trajectory as does a calculation based on direct methods employing forces and accelerations.

Although Euler had made a substantial beginning, he did not continue with his research on variational principles; his ideas remained promising suggestions rather than fully developed concepts. The elaboration of the theory was achieved in the decades that followed by his younger contemporary Lagrange. In his first contribution to the subject in 1762, Lagrange employed the new δ-process that he had introduced into the calculus of variations (§3.5). Beginning with a more general form of Euler's principle of least action, he showed how it led to a uniform procedure for generating the equations of motion of an arbitrary dynamical system (Fraser *1983*).

Despite his considerable success with this principle, Lagrange proceeded in the next few years to replace it with another law, derived from a dynamical generalization of the principle of virtual velocities. His shift in approach was influenced by technical considerations, by a desire for a unified treatment of both statics and dynamics, and by an aversion to the metaphysical associations of least action. The new formulation became the basis of his *Méchanique analitique*, published in 1788 in Paris when he was 52 years old. He began with the fundamental axiom

$$\sum m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}\cdot\delta\mathbf{r} = \sum \mathbf{F}\cdot\delta\mathbf{r}, \tag{5}$$

where m is the mass of a typical body, $\mathbf{v}$ is its velocity, $\delta\mathbf{r}$ is its virtual displacement and $\mathbf{F}$ is the applied force acting on it. Given that the system is described in terms of a set of 'generalized' coordinates $(q_1, q_2, q_3, \ldots, q_n)$, he derived the 'Lagrangian' equations of motion

$$\frac{\partial T}{\partial q_i} - \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial \Phi}{\partial q_i} = 0, \tag{6}$$

where T denotes half the live force, and Φ is the potential.

The *Méchanique analitique*, published a century after Newton's *Principia*, was the most developed expression of the eighteenth-century abstract analytical tendency in mechanics. Lagrange's variational approach was most important in advancing new methods, in mathematicizing the study of dynamical systems, rather than in enlarging our understanding of physical phenomena. He provided a uniform procedure for generating the equations of motion that was independent of the coordinatization employed or of any assumption concerning the material constitution of bodies. A conspicuous feature of his approach was his use of constraints to simplify mathematically the description of the system. The notion of a constraint enabled one to idealize the analysis in a way that avoided detailed assumptions concerning the physical basis of the phenomena. Formal mathematical development replaced physical hypothesis and experimental verification.

5 HAMILTON–JACOBI THEORY

The legacy of Lagrange was continued in the nineteenth century in the development of what is known in modern physics as 'Hamilton–Jacobi theory', the set of methods for formulating and integrating the differential equations of motion of a general dynamical system. The theory was established by the young Irish scientist William Rowan Hamilton in two papers published in the Royal Society's *Philosophical Transactions* in 1834 and 1835. These researches originated in his investigation of questions in mathematical optics and in his study of the three-body problem in celestial dynamics (§8.9). Two years later, Carl Jacobi introduced important modifications and additions to Hamilton's work in a paper that appeared in Crelle's *Journal*.

By means of variational methods, Hamilton showed that Lagrange's system (6), a set of n second-order differential equations, could be replaced by a new system consisting of $2n$ first-order equations, the so-called 'canonical differential equations of motion'. He did so by expressing (6) in terms of the q_i and an additional set of variables, the 'conjugate momenta' $p_i = \partial T/\partial \dot{q}_i$. In his further study of a problem of perturbation, he employed a 'contact transformation' – that is, a change of variables that preserved the form of the canonical equations. Jacobi subsequently developed a systematic theory of contact transformations and reduced the problem of integration to the solution of a single partial differential equation containing a principal or generating function (Prange *1933*).

In the later nineteenth century the methods of Hamilton and Jacobi were investigated by such people as Joseph Liouville and Rudolph Lipschitz as part of the further development of mathematical mechanics and differential geometry. In addition to their purely theoretical interest, these methods were important in applications to celestial mechanics (§8.8). In the 1920s, Schrödinger and Louis de Broglie showed that they also provided an appropriate formalism for expressing the new quantum-wave physics. As a consequence of its role in quantum mechanics, the theory of Hamilton and Jacobi occupies a rather more prominent place today than its historical origins and position in the classical subject (as well perhaps as its intrinsic interest) might otherwise indicate.

6 CONTINUOUS MEDIA

Newton achieved only special (albeit significant) results in his investigation of fluids in Book II of the *Principia*, and he contributed little to the study of flexible and elastic bodies. Much effort in the period 1700–1830 was devoted to constructing an adequate mathematical theory for these materials. These investigations had their origins in the work of such early mechanicians as Huygens, James I Bernoulli and Antoine Parent, and were continued and brought to maturity by mathematicians Euler, d'Alembert, Alexis Clairaut, Lagrange, Augustin Louis Cauchy, and many others. The resulting body of results makes up a substantial part of modern civil and mechanical engineering.

Theories of continuous media developed from the study of such special problems as the shape of a suspended cable, the deflection of an elastic beam, the strength of struts and columns, the vibration of a taut string and the resistance of bodies in fluids. Each of these problems required its own concepts and techniques of solution. The progress of the subject abounded with false starts, long detours and brilliant successes.

The analysis of elastic beams provides an illustrative study in the history of continuum mechanics (see §8.7 on the theory of structures). Between 1695 and 1705, James I Bernoulli wrote three seminal papers analysing the deformation of an elastic blade or lamina subject to forces acting at its ends. He considered a lamina built into a support at one end and loaded at the other end by a weight (Figure 2); the problem was to determine the curve of deflection. By considering the structure of the deformed lamina at a cross-section, he concluded that the tensile force on the fibres was inversely proportional to the radius of curvature of the curve of deflection at its intersection with the cross-section. The force would exert an internal resisting moment that in static equilibrium would just balance the moment due to the external load. Let the lamina be described in a Cartesian (x, y) coordinate system in which the load is situated at the origin, the y-axis coincides with the line of action of the load, and the coordinates of an arbitrary point of the lamina are x and y. The calculus, for Bernoulli a very new and exciting mathematical tool, gave the expression $r = -\mathrm{d}^2y/\mathrm{d}x\,\mathrm{d}s$ (where $\mathrm{d}s^2 = \mathrm{d}x^2 + \mathrm{d}y^2$) for the radius of curvature. Using the condition that the moments balance, he obtained a differential equation to describe the static configuration of the elastica (the function representing the elastic line curved in the plane):

$$-k\frac{\mathrm{d}^2y}{\mathrm{d}x\,\mathrm{d}s} = Px. \tag{7}$$

He considered the case where the load acts perpendicularly to the lamina,

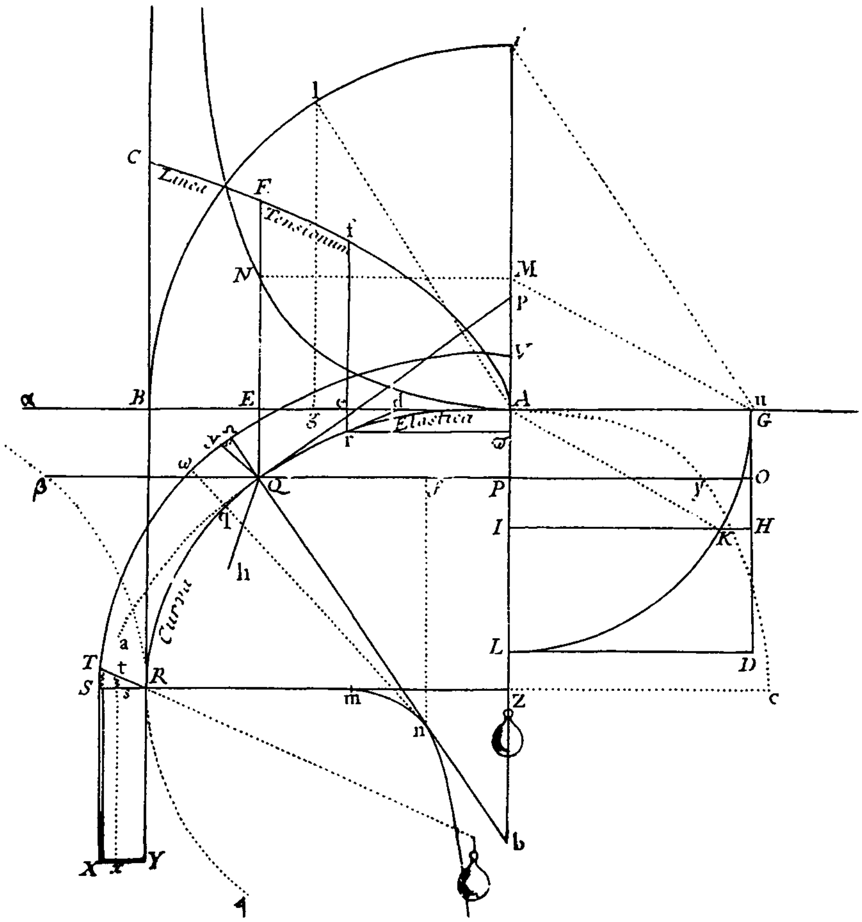

Figure 2 The deformation of an elastic lamina, from James I Bernoulli's *Curvatura laminae elasticae* (1694)

the so-called 'rectangular elastica', and integrated equation (7) once to obtain a first-order differential equation connecting x and y. (In later mathematics its solution would be called an elliptic integral; see §4.5.) James I Bernoulli introduced infinite series to investigate this equation, and discussed some of its properties.

In 1744 Euler, then a 37-year-old academician in Berlin, published an important mathematical analysis of the elastica. He took equation (7) and completed James I Bernoulli's investigation by enumerating all the forms assumed by an elastic lamina loaded at its ends. He also considered the application of the theory to the study of the strength of columns. An elastica that is subject to external terminal forces making small angles with its long axis provides a suitable model for a column undergoing compression. Euler derived an expression (known later as 'Euler's buckling formula') giving the minimal or critical load necessary to bend the column. In 1779 Lagrange extended his results using elliptic integrals, and showed that higher-order configurations are associated with a larger value for the critical load and therefore lead to a substantial strengthening of the column.

The work of James I Bernoulli, Euler and Lagrange was carried out in an atmosphere of intellectual excitement created by the power of the new analytical methods to solve physical problems. At this time, mathematical sophistication often ran far ahead of physical understanding. A weak point in everyone's treatment of the elastica concerned the constant k in equation (7). This quantity incorporated all the information about the individual structure and material elasticity of the lamina, and was simply introduced – on no certain grounds – in order to derive equation (7). A satisfactory theory would need a detailed analysis of the three-dimensional structure of the lamina in terms of the concepts of neutral axis, moment of inertia, and tensile and compressive elastic stress. It was only very slowly and with great difficulty that such a theory was forthcoming.

Although one can find during the eighteenth century significant special anticipations of modern stress analysis, the general concept of elastic stress itself never emerged. A developed mathematical theory of continuous media first appeared in the 1820s, in the writings of Cauchy and Claude Navier. Working independently of Cauchy, Navier employed the idea of elastic stress to construct a complete analysis of the elastica, published in 1826 in his *Resumé des leçons données à l'Ecole des Ponts et Chaussées*. Beginning with the results of the late-eighteenth-century scientist Charles Coulomb, he obtained the modern formula, EI/R, for the resisting moment of a section of a loaded beam, where E is the modulus of elasticity, I is the moment of inertia of the section about the neutral axis, and R is the radius of curvature of the curve of deflection at the point where it passes through the section.

In 1829 Cauchy stipulated generally that, given any surface S on or within a body that bounds a part V of this body, we may assume that the matter exterior to V exerts a field of stresses or forces per unit area on S. Using this seminal idea, together with the principles of linear momentum and moment of momentum, he developed an analytical theory applicable to elastic materials and fluids. Modern continuum mechanics was born with his treatises of the 1820s.

For a more detailed survey of elasticity theory, see §8.6.

7 MECHANICS IN FRANCE, 1788–1830

Navier worked in mechanics at a time when Paris dominated world science. Along with fellow engineers Jean Victor Poncelet and Gustave Coriolis, he graduated from the Ecole Polytechnique, founded in 1795, and with them participated in the extraordinary scientific vitality of this school in its first decades. The three men built on the researches of such earlier figures as Bernard Bélidor, Coulomb, Lazare Carnot and Gaspard de Prony. Their distinctive achievement was to combine the highly theoretical programme of mathematical mechanics of Euler and Lagrange with the more practical tradition of civil and mechanical engineering long established in France. The Ecole Polytechnique and the Ecole des Ponts et Chaussées played a prominent role in focusing and coordinating their efforts, in which research concerns had strong links to engineering education. As historian I. Grattan-Guinness has noted (*1984*: 31), the story here is 'an extraordinary *potage* of mathematics, mechanics, engineering, education and social change'.

The development of the concept of mechanical work illustrates the achievements of the French engineers. The principle of live force (equation (3)) was a well-established relation of theoretical mechanics by the middle of the eighteenth century. Although in later physics it would become known as the conservation of mechanical energy, there was at this time little recognition of the concepts of work and kinetic energy, and of their interchangeability. The principle served largely as a mathematical relation that imposed an analytical condition on the motion of the system.

In 1783 Lazare Carnot, then an unknown 30-year-old French engineer, published a treatise on efficiency in machines. He introduced the term 'moment of activity' for force times distance, and identified it as the determinative quantity in analysing the performance of weight-driven machines. Beginning with the principle of virtual velocities, he derived a relation connecting moment of activity and live force. He concluded that to obtain the maximum 'effect' or work from a machine it was necessary to avoid inelastic shocks in its parts. In a first primitive recognition of the interconvertibility of kinetic energy and work, he observed that such shocks would reduce the total live force and hence also the work available (Gillispie *1971*).

In 1819 Navier edited a reprinting of Bélidor's *Architecture hydraulique* (1737) in which he called attention to Carnot's researches on machine efficiency. With the rapid development of industrial technology in France during this period, there was a strong interest in the theoretical analysis of machines. In 1829 Coriolis and Poncelet published books in which they introduced the term 'work' for the product of force and distance. Coriolis explicitly redefined the live force of a mechanical system as $\frac{1}{2}\Sigma mv^2$ (sometimes called the 'reformed live force'), where the factor $\frac{1}{2}$ constituted explicit theoretical recognition of the primary place of the concept of work.

8 PHILOSOPHICAL CURRENTS

Since Newton, philosophical questions about the nature and meaning of classical mechanics have intruded in the development of the subject. Although these discussions have often concerned general methodological assumptions unconnected to technical work in the field, no historical survey would be entirely complete without some consideration of them.

Throughout the history of mechanics there has been an opposition between an empirical, phenomenalist approach to the world, and an approach that emphasizes theory and going beyond the appearances. Newton wrote in the *Principia* that he 'frames no hypotheses' to defend his refusal to seek a physical basis for gravitational interaction. Whereas he believed that systematic mathematical analysis was sufficient to account for the observable phenomena, the Cartesians sought a more fundamental explanation that would interpret the world from first principles in terms of primitive concepts of extended matter in motion. The Cartesians accepted Newtonian dynamics, but denied that it provided an ultimate explanation of physical reality.

This tension reappears in a different form in the early nineteenth century in the opposition between Lagrange's analytical mechanics and Siméon Denis Poisson's 'physical mechanics' (as he called it). Poisson believed that Lagrange's theory involved too severe an idealization and abstraction of physical reality, that it was necessary to approach nature at a deeper level in terms of molecular models of the structure of matter. He wanted to replace the notion of constraint and the assumption of perfectly hard bodies by definite molecular mechanisms. Although his attitude to physical theory placed him within the contemporary programme of Laplacian physics, it set him at odds with the influential mathematical positivism of his countryman Joseph Fourier, and led to his progressive isolation within French science.

The dialectic between phenomenology and realism arises at the end of the nineteenth century in the writings of Ernst Mach. His *Die Mechanik in ihrer Entwicklung historisch–kritisch dargestellt* (1883) was one of several treatises of the period devoted to a critical study of the foundations of mechanics. Along with John Stallo and Pierre Duhem, he was a leading *fin-de-siècle* proponent of positivistic physics, and criticized what he regarded as the excessive employment of atomic–mechanical hypotheses. He believed that phenomenological concepts such as energy were more likely to reveal the inner character and unity of physical theory. His philosophy of science led him to oppose Ludwig Boltzmann's statistical mechanics (§9.14) and increasingly placed him at odds with prominent trends in contemporary physics.

Positivistic philosophy can result in a sterile and limited conception of physical theory and its possibilities. There are nevertheless two respects in which it has had a beneficial influence in the history of science. In granting each theory autonomy on the basis of its individual phenomenological domain, it avoids a rigid mechanistic reductionism and leads to a principle of tolerance in the development of new theories.

In emphasizing observation, empiricism necessarily takes note of the role of the observer in the description of a physical system. Newton's insistence on the absolute character of space and time contradicted his own empiricist refusal to frame hypotheses, and it was natural of Mach to have rejected this part of Newton's mechanical philosophy. Mach's positivism prepared the way for the advent of relativistic conceptions in the work of Henri Poincaré and Einstein. Einstein had read Mach, and the latter's influence is apparent in Einstein's fundamental recognition in special relativity that it is necessary to incorporate the observer into the description of a physical system.

With the establishment of special relativity (§9.13), it became necessary to introduce the adjective 'classical' to delineate the vast range of mechanical doctrines from Newton to Einstein. Classical theories retain their validity and continue to be cultivated extensively today in mathematical engineering. Nevertheless, since Einstein, the classical viewpoint has lost its epistemological primacy as a final description of material motion in space and time.

9 POSTSCRIPT: ON THE TERM 'RATIONAL MECHANICS'

In the preface to the *Principia*, Newton wrote of 'rational mechanics' to refer to the study of motions engendered by forces and, conversely, the study of forces that correspond to given motions. The adjective 'rational' was used to distinguish this subject from practical or common mechanics, and served to emphasize the abstract, general character of his investigation.

The term 'rational mechanics' was not employed as a formal category by the leading mathematical scientists of the eighteenth century. Although d'Alembert mentioned Newton's terminology in his article 'Méchanique' in Volume 10 of the *Encyclopédie* (1765), *mécanique rationelle* (sic) was not included as a subject classification and did not appear in the famous 'Systême figurée des connoissances humaines' at the beginning of the work. Neither was it in his *Traité de dynamique* (1743).

August Comte adopted the term *mécanique rationnelle* in his *Cours de philosophie positive* (1830 onwards). Comte recognized the basis of mechanics in experience, but he also wished to emphasize the theoretical character of the subject and felt that the adjective 'rational' was well suited to do that. He would in fact have preferred the term *phoronomie*, originally introduced by Jacob Hermann in 1716.

As a result of Comte's influence, the designation *mécanique rationnelle* was commonly used in France in the nineteenth century to refer to theoretical mechanics. It seems, however, not to have gained much currency in Britain, the USA or Germany. It is not in the major English, American and German dictionaries. In Harrap's French–English dictionary, *mécanique rationnelle* is translated as 'theoretic mechanics, pure mechanics'. The term has been introduced more recently by Clifford Truesdell, who uses it in his historical and scientific writings to identify a mathematically rigorous, deductive approach to mechanics.

In discussing the eighteenth century, the term 'rational mechanics' as used by Newton should be distinguished from Cartesian rationalism. Hankins *1970*, Gaukroger *1982* and Pulte *1989* have documented the influence of Cartesianism on the physical science of the period, referring however to a definite set of philosophical attitudes with specific historical origins. While one can recognize Cartesian echoes in Newton's usage, he was not expressing a formal philosophical doctrine and it would be premature to infer a deeper meaning in his terminology. In view of the lack of usage by scientists of the period, its rareness in modern writing outside France, and the possibly misleading associations with Cartesian rationalism, it seems preferable not to employ the term 'rational mechanics' for theoretical mechanics in the eighteenth century.

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