Results from MathSciNet: *Mathematical Reviews* on the Web
© Copyright American Mathematical Society 2026

**MR4960087** 01A50 01A45 12-03 51-03

**Charmillon, Jean-Louis** (E-POFA-ECB); **Viader, Pelegrí** (E-POFA-ECB)

**The algebraic proofs of Cotes' factorization theorem: Newton, Bernoulli, and De Moivre. (English. English summary)**

*Br. J. Hist. Math.* **40** (2025), *no.* 2, 83–122.

The subject of this article is a result stated by the British mathematician Roger Cotes (1682–1716) and published posthumously in 1720. Cotes was interested in factorization of polynomials of the form $x^n \pm a^n$. This interest was motivated by the use of partial fractions to integrate rational functions. Cotes ascertained geometrically (without a general proof) that these polynomials could be expressed as the product of linear and quadratic factors, a result that would be given a century later by the fundamental theorem of algebra. The geometric work of Cotes and others is little known today and is a fascinating subject area in the history of early eighteenth-century mathematics.

A proof of Cotes' theorem was presented in 1730 in a book by Abraham De Moivre. In 1742 Johann Bernoulli published work on polynomial factorization presenting research that he had carried out fifteen to twenty years earlier. The article under review describes the work of Johann Bernoulli and De Moivre in some detail and also includes an examination of some unpublished writings of Isaac Newton from the period 1674–1684 (most probably in 1676–1677). There is as well a fair degree of technical exposition that goes beyond the history of mathematics strictly speaking, but the paper offers interesting mathematical perspectives on factorization. In contrast to Cotes' geometric formulation involving circle lines, the approaches of Johann Bernoulli, De Moivre and Newton were algebraic in character.

The authors present a slightly modernized treatment of the original material. For example, trigonometric functions are used in the account of De Moivre's 1730 book, although such functions did not enter mathematics until a decade later in early work of Leonhard Euler [see V. J. Katz, Historia Math. **14** (1987), no. 4, 311–324; MR0919063]. De Moivre's complex identity is presented in its standard modern trigonometric form. The writings of Newton taken up for study were not published until 2008 and played no role in the history of factorization.

The authors introduce the terminology of "incomplete induction" to describe inferences that are based on the identification of a pattern but do not include an actual proof. Such inferences appear in Johann Bernoulli's writings on Cotes' theorem. One might take exception with the authors' more general suggestion that the forms of justification found at the time—some of them mathematically substantial—served a primarily psychological role in the logic of the investigation.

The extra-historical mathematical forays into Cotes' theorem in Section 7 are fairly detailed and together with the result in Section 4.2 and the appendix occupy a third of the whole article. In Section 7 the authors investigate a method that involves dividing $x^n$ by the quadratic expression $x^2 - ax + 1$ and analyzing the remainder term $R_n(x, a)$ in terms of something called Chebyshev polynomials (named after the Russian mathematician Pafnuty Chebyshev (1821–1894)). Relevant results concerning these polynomials are derived in the appendix. The authors' account here follows extensively the first two chapters of [J. C. Mason and D. C. Handscomb, *Chebyshev polynomials*, Chapman & Hall/CRC, Boca Raton, FL, 2003; MR1937591].

The authors believe that, in addition to its mathematical interest, Section 7 fosters historical understanding (p. 83): " … our own reconstruction of a proof using only the methods available to these mathematicians raises compelling questions about why they did not pursue similar strategies, thus opening new avenues for historical and mathematical investigation." The theory developed in Section 7 and the appendix may strike the reader as somewhat complicated and it is not immediately clear that there is

Results from MathSciNet: *Mathematical Reviews* on the Web
© Copyright American Mathematical Society 2026

a strong affinity between it and the work of Newton, Johann Bernoulli and De Moivre. Nevertheless, in keeping with the modernizing sensibility of the article, the authors take their main result and examine from this angle the work of the three mathematicians. They conclude that Newton came closest to anticipating the insights contained in their reconstruction. *Craig G. Fraser*