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A History of Vector Analysis. The Evolution of the Idea of a Vectorial System

Michael J. Crowe; Craig G. Fraser

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Robert H. Romer, *Editor*
Department of Physics, Amherst College, Amherst, Massachussetts 01002

A History of Vector Analysis. The Evolution of the Idea of a Vectorial System. Michael J. Crowe. 270 pp. Dover Publications, New York, 1985. Price: $7.00 (paper). (Reviewed by Craig G. Fraser.)

Vector analysis is that branch of mathematics that studies directed magnitude using algebra and calculus. The prehistory of the subject may be traced to the early 1600s when Pierre Fermat and René Descartes applied François Viète's new symbolic operational algebra to the investigation of plane curves. Vector analysis proper began in the mid-18th century with Leonhard Euler's introduction of orthogonal coordinate systems into analytic geometry. Euler used the analytical description of directed geometrical quantities to study curves and surfaces in space.

Vector analysis in its modern form developed in the 19th century from the work of William Rowan Hamilton [*Lectures on Quaternions* (1853)] and Hermann Grassmann [*Theory of Linear Extension* (1844)]. In the 1880s and 1890s the physicists J. Willard Gibbs and Oliver Heaviside, working partly from these early sources and partly independently, devised the familiar vectorial notational system used today in physics and advanced calculus.

Michael J. Crowe's *A History of Vector Analysis*, originally published in 1967 and now rereleased in paperback by Dover, is a readable, nontechnical account of the 19th-century history leading up to the consolidation of the Gibbs–Heaviside system of notation. Crowe describes the invention by Grassmann and Hamilton of their respective algebraic systems for the representation and investigation of directed geometrical quantity. He then turns to the reception of their work and discusses the early researches of Gibbs and Heaviside. A chapter is devoted to the "struggle for existence" waged in the 1880s and 1890s between supporters of the two systems, a struggle which saw proponents of Hamilton's quaternions increasingly isolated and defensive. The conclusion documents the adoption of the Gibbs–Heaviside system (in effect, a victory for Grassmann) in physics textbooks at the beginning of this century.

In Crowe's history, important issues are not addressed. Gibbs wrote in 1893 that "the notions which we use in vector analysis are those which he who reads between the lines will meet on every page of the great masters of analysis." (Crowe, p. 199) Did Gibbs' vector analysis incorporate conceptual or mathematical advances, or was it just a notationally convenient formulation of Cartesian methods and results? In modern mathematics, the concepts of vector and vector space are introduced in linear algebra. What is the relation, if any, between the researches of Hamilton and Grassmann described by Crowe and the contemporary 19th-century work, mathematically very substantive, in linear and abstract algebra?

In the Preface to the Dover edition, Crowe provides a summary of the secondary historical literature that has appeared in the last two decades. This literature deals with technical and historical questions not discussed by him and forms a necessary supplement to his book.

Craig G. Fraser received his Ph.D. in the history of science from the University of Toronto and did postdoctoral work at Princeton and Yale. He is currently assistant professor for the history of mathematics at the Institute for the History and Philosophy of Science and Technology of the University of Toronto. His research activities center on the history of mathematical analysis in the 18th and 19th centuries.

Was Einstein Right? Clifford M. Will. 274 pp. Basic Books, New York, 1986. Price: $18.95. (Reviewed by Hans C. Ohanian.)

Enticed by the title, some readers will expect this book to tell them what is wrong with Einstein's theory or, at least, what could be improved. Such readers will be disappointed—*Was Einstein Right?* deals with the more mundane, and more worthwhile, business of recent experimental tests of general relativity.

The last 20 years have seen remarkable progress in experimental relativity. Until 1960, the only confrontation of general relativity with experiment and observation consisted of the three "classical" tests: the perihelion precession, the light deflection, and some qualitative observations on the red shift. Now, thanks to the development of new, precise experimental and observational techniques, we have been able to test general relativity in several other ways: direct measurements of time dilation, deflection of radio waves, retardation of radio waves, solar oblateness, lunar laser ranging, and the indirect observations of the energy loss produced by gravitational waves in the binary pulsar. These tests have brought about what Will designates a Renaissance of general relativity. It is an apt designation, and we might regard the 50 or so years before this Renaissance as the Middle Ages, during which relativistic learning was kept alive by a few mathematical monks scribbling in isolation in their monasteries.