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★The creation of continuous exponents: a study of the methods and epistemology of John Wallis.

Research in collegiate mathematics education, II, 33–60, CBMS Issues Math. Ed., 6, Amer. Math. Soc., Providence, RI, 1996.

The subject of the article is connected to issues in mathematics education that are stimulated in part by Imre Lakatos' philosophical writings. Lakatos wanted to look beyond the logical formalist presentation of mathematics and to uncover the underlying methodology whereby new mathematics is actually produced. His concept of rational reconstruction, of idealized historical case studies, was employed as a critical tool to accomplish this goal. Although Dennis and Confrey are sympathetic to Lakatos' position, they favour a more fundamental place for history in understanding mathematics, as a forum for what they term "multiple forms of representation". They contrast the logical formalist approach of modern mathematics with what they characterize as the more practical and "empirical" nature of mathematics in the seventeenth and eighteenth centuries. The core of the paper is an account of Wallis' Arithmetica infinitorum (1655), including a very nice exposition of his original derivation of the famous Wallis formula for $\pi/2$.

In the conclusion, the authors observe that "The methods of investigation outlined in this paper have been largely purged from our mathematics curriculum. These mathematical results are now presented to students in a formal logical setting that came about in the nineteenth and twentieth centuries." (The modern derivation of Wallis' formula, not given in the paper, involves recursive expressions for the integral of $\sin^{2n} \theta$ on the interval from 0 to $\pi/2$.) They argue persuasively that a student of mathematics can learn much from a study of the older methods and results.

Historians may take exception to the characterization of mathematics in the period 1600-1800 as empirical/practical and mathematics after 1800 as logical/formalist. There were definite rational principles and points of view in the older period, and there are also pragmatic elements in modern mathematics. On the other hand, the idea of multiple forms of representation does seem a very useful concept, and points to the existence of a deep underlying epistemological relativism in the historical development of mathematics. Craig G. Fraser