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★**Formal versus convergent power series.**

Mechanics, analysis and geometry: 200 *years after Lagrange*, 549–558, *North-Holland Delta Ser.*, *North-Holland*, *Amsterdam*, 1991.

This is not a historical article in the usual sense (there are no references to either primary or secondary sources). The author's point is to show that certain computations which Euler carried out on infinite series can be justified if one recalls that "the ring of formal power series with complex coefficients has a *topology*" (emphasis in original). This claim is illustrated with some examples. Thus Euler set

$$\left(1+\frac{x}{n}\right)^n (\text{“}for\ n=\infty\text{”}) = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots, \tag{1}$$

which he justified by noting that

$$\binom{n}{p}\left(\frac{x}{n}\right)^p = \frac{1}{p!}\frac{n(n-1)\cdots(n-p+1)}{n^p}x^p \tag{2}$$

"is equal to $x^p/p!$ for $n=\infty$". Interpreted in the ring of formal power series with complex coefficients, equation (1) says that "the sequence of polynomials $(1+x/n)^n$ (considered as formal power series with only a finite number of nonvanishing coefficients) converges to the formal power series $1+x/1!+x^2/2!+\cdots+x^n/n!+\cdots$". The proposed interpretation, it is suggested, exonerates Euler's analysis from the criticism (made by Nicholas Bernoulli and 19th-century analysts) that it is deficient in rigor.

The problem with this article is that neither Nicholas Bernoulli nor the 19th-century analysts would have had any objection to the reasoning by which Euler obtained (1). Rather they questioned certain subsequent steps which seemed to require convergence for their validity. The particular interpretation in terms of formal power series proposed by the author applies to that part of Euler's analysis which is unproblematic, and which has always been regarded as unproblematic. It offers only slight mathematical illumination, and no historical clarification of his work. *Craig G. Fraser*