Review

Reviewed Work(s): Families of Curves and the Origins of Partial Differentiation by Steven B. Engelsman

Review by: Craig Fraser

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common theme it is that scientists are human beings and should be understood in terms of human processes: not much to ask in principle, perhaps, but an awful lot in practice.

JOHN HENDRY
London Business School

STEVEN B. ENGELSMAN. **Families of Curves and the Origins of Partial Differentiation.** Amsterdam: North-Holland, 1984. Pp. x+238. ISBN 0-444-86897-6. US $29.00, Dfl. 85.00.

Partial differentiation is presented today in mathematical analysis as part of a theory of functional mappings defined on domains of real numbers. One considers a function $z = f(x,y)$, defined on some region of the real plane and interpreted geometrically as the equation of a surface in three space, and considers the partial derivatives of z that result when x and y are successively held constant. These derivatives are real-valued mappings obtained in the usual way as the limits of Newton quotients.

Steven B. Engelsman shows that, historically, partial differentiation developed out of a different class of problems, namely the problem of constructing orthogonal trajectories to a family of curves. In the early eighteenth-century, partial differentiation was known as 'differentiation from curve to curve'. The goal of each investigation was the derivation of a differential relation to describe the given orthogonal trajectory. Only later, with the work of d'Alembert in the 1740s, did the focus of research shift to the problem of integrating partial differential equations.

The study of partial differentiation as it emerged in the early eighteenth century was conceptually different from today's subject. The Leibnizian calculus consisted of an algebraic theory that was interpreted geometrically. The algebraic variable was the central concept of the calculus, and the curve was the primary object of study. Theorems of analysis were regarded as true because of the formal correctness of the underlying algebra, and because this algebra received a clear interpretation in the geometry of curves. The concept of functional mapping, questions of domain and the distinction between local and global behaviour involve modern notions foreign to the world of eighteenth century analysis.

Engelsman provides a detailed and careful guide to the work on partial differentiation between 1695 and 1740 of Leibniz, Johann Bernoulli, Jakob Bernoulli, Nicolaus I. Bernoulli and Euler. Highlights of his study include the following: the original presentation of Leibniz's rule on the interchangeability of differentiation and integration (pp. 43–46); Nicolaus I. Bernoulli's investigation of partial and total differentiation (pp. 92–112); Euler's proof of the equality of mixed second order differentials (pp. 128–130) and the discussion of the coefficient lemma for total differentials (pp. 147–149). All quotations from foreign languages are translated into English; the original passages (in Latin, French and German) are given in endnotes. Appendix Two consists of the text and English translation of a previously unpublished memoir by Euler from the early 1730s on the differentiation of functions of two or more variables. The book closes with a bibliography of primary and secondary sources. There is no index.

CRAIG FRASER
University of Toronto

Editorial note: We regret that, by an editorial oversight, the above review appeared in volume 19, part 3 of *BJHS* (November 1986) wrongly attributed to Jeremy Gray. We apologize to all concerned.