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Newton's first inverse solutions.
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This paper discusses Problems 6 and 7 of Newton's draft paper on dynamics, *De motu*, of 1684. Problem 6 concerns a body that moves in a straight line through a medium and is subject to a resisting force proportional to the speed. It is required to find the relation between time and speed and distance traveled. Newton deduced that, for successively equal time intervals, the corresponding speeds are in a geometric progression. Using an orthogonal coordinate system he concluded that, if the distance and speed are measured along the horizontal axis, then the time will be given by the area under a rectangular hyperbola. In Problem 7 he extended this result to the case where a constant gravitational force is assumed to act on the body. In a scholium he considered the problem of projectile (curved) motion in the medium. In his published *Principia mathematica* of 1687 these results became Propositions 1–4 of Book 2.

The paper follows an interesting earlier article of the author [Centaurus **34** (1991), no. 3, 272–283; MR1166074]. In the latter he analyzed Newton's demonstration in the *Principia* of Proposition 2 of Book 2. The particular proof structure of this proposition was described apparently for the first time—and shown to be similar to the one employed by Newton in Proposition 1 of Book 1. In the present paper the author gives a reconstruction of the reasoning involved in Newton's earlier solutions in *De motu*. He regards the solution of Problem 6 to be significantly different from the one in the *Principia*. He notes that Newton did not use integration to solve Problem 6. He writes: "until one has actually seen Newton's solutions one has to wonder what approach anybody could possibly use other than integration". He suggests that Newton reasoned backward from the case of the hyperbola, in a procedure involving the "matching" of mathematical and physical situations.

Taken together the two articles provide an informative study of how calculus and dynamical processes were understood by Newton. It is, however, possible to take exception with several of the points made by the author. Thus the derivation in the *Principia* looks very much like a more detailed version of the one in *De motu* in which the basic idea is made more explicit. To assert that there is "really no physics" in the *De motu* derivation seems questionable. (The author's own views seem to have changed. In the earlier article he stressed the mathematical character of the *De motu* solution of Problem 6; in the paper under review he criticizes Herivel for suggesting that its interest is largely mathematical.) The suggestion that Newton was proceeding in Problem 6 by matching mathematical and physical situations seems speculative. Newton does employ an integration in Problem 6 to get the distance in terms of the time, and he also employs integration in Problem 7. Finally, it should be noted that the argument that Newton develops in Proposition 2 of Book 2 of the *Principia* can be understood as a purely mathematical one. It enables him to obtain directly (via approximation and passage to the limit) an exponential solution of the differential equation $dy/dx = ky$ without having to bring in any properties of the integral $x + a = \int_b^y dy/ky$. *Craig G. Fraser*