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An approximation technique, and its use by Wallis and Taylor.
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Chapter 9 of the author's *The mathematics of Plato's academy* [Oxford Univ. Press, New York, 1987; MR0932963] consisted of a selected historical survey of research on continued fractions. The article under review is a continuation of that study. Described are work of John Wallis in his *Treatise of algebra* (1685) on an approximation of pi and a short paper of Brook Taylor's of 1717 presenting a method for approximating $\log_{10} 2$. The author's aim is "to see what kind of understanding emerges from what is called, in literary criticism, a 'close reading' or explication de texte of these two passages by Wallis and Taylor, and to place them in their immediate mathematical and historical context".

The approximation technique referred to in the title is the following. Assume x is the real number to be approximated and suppose $a/b < x < c/d$. Then $a/b < (a+c)/(b+d) < c/d$. Hence we obtain a new approximation to x: either $(a+c)/(b+d) = x$, $(a+c)/(b+d) < x < c/d$ or $a/b < x < (a+c)/(b+d)$. By successively considering the quantities $(a+c)/(b+d)$ we obtain a sequence of values p_i/q_i that approximate x. It turns out that this process is connected to the quotients that result when we apply the Euclidean algorithm to x and 1. Thus the number of consecutive underestimates or consecutive overestimates in the process is equal successively to the quotients yielded by the Euclidean algorithm applied to x and 1.

Both Wallis and Taylor employed procedures that involve a form of the approximation process just described, although neither seems to have been aware of the connection with the Euclidean algorithm and continued fractions. Wallis's method was based on finding approximations by observing the behaviour of the fractional parts of a multiplication table. Taylor's procedure was based on an considering successive approximate values to x in the equation $2 = 10^x$.

Wallis published an initial account of his method in 1678 in an appendix to the second edition of Jeremiah Horrock's *Opera posthuma*, and presented a longer treatment in Chapters 10 and 11 of his *Treatise of algebra*. His method is not always easy to follow, in no small part because of what the author calls Wallis's "contorted and verbose explanations". The greater part of the article is devoted to a detailed analysis of his procedure.

Wallis was the first to use the term "continued fraction", in his *Arithmetica infinitorum* of 1665. It is therefore surprising that, contrary to what some commentators have concluded, he apparently failed to recognize the connection of his method and the process arising from the Euclidean algorithm. The author suggests two reasons for this. Wallis's research on continued fractions centred on general continued fractions, which are not clearly related to the Euclidean algorithm. Second, in his work on approximation he did not engage in the sort of refinement and further investigation that would have revealed the connection between his approximation procedure and this algorithm.

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