Mathematics in Library Subject Classification Systems

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Abstract Insofar as library science is concerned, modern classification of mathematical subjects occurred within the larger framework of library classification, a vast project receiving sustained attention in the period from 1870 to 1920. The work of the library cataloguers was carried out against the background of a broad nineteenth-century interest in the classification of knowledge. We explore different views during this period concerning the position of mathematics in the overall scheme of knowledge, the scope of mathematics, and the internal organization of the different parts of mathematics. We examine how mathematical books were classified, from the most general level down to the level of particular subject areas in analysis. The focus is on the Library of Congress classification system in its various iterations from 1905 to the present.

Keywords Library classification • Mathematical books • Analysis • Library of congress

1 Introduction

> The classification of mathematical studies is involved in extraordinary difficulties, and so is the classifying of many mathematical books. The relations of the branches are so intricate, so plastic, so recondite, that it is well-nigh impossible to define them or to comprehend them. - Bliss (1935, 20)

Insofar as library science is concerned, classification of mathematical subjects occurred within the larger framework of library classification, a vast project which drew sustained attention between 1870 and 1920. The two American giants in library work in the formative period of classification were Melvil Dewey and Charles Cutter. In 1876 Dewey published the famous Dewey decimal system of classification, while Cutter's expansive scheme of 1885 would provide the basis

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M. Zack, D. Schlimm (eds.), *Research in History and Philosophy of Mathematics*, Proceedings of the Canadian Society for History and Philosophy of Mathematics
La Société Canadienne d'Histoire et de Philosophie des Mathématiques,
DOI 10.1007/978-3-319-64551-3_12

for the Library of Congress system. The latter was established in 1905 by James Hanson and Charles Martel, both European immigrants to the United States. In the early twentieth-century additional classification schemes appeared. Among the more notable of these were the subject "system" formulated by the Englishman James Duff Brown and the "bibliographic" system invented by City College of New York librarian Henry E. Bliss.

The practical goal of all classification was information retrieval, allowing, for example, a user to go to a large library, consult the catalogue, and retrieve a given book of interest. The call number given to a book had to be abstract—it could make no reference to any particular library or to the physical arrangement of the books in any library it happened to belong to.[1] The motivation for classification schemes was the appearance of an increasing number of comprehensive libraries with substantial holdings of books from many subject areas. Library science, as the discipline of book classification and cataloguing had come to be known by the 1920s, was not a fundamental science, but an organized and systematic endeavor with the goal of identifying and retrieving information. Of course, librarians were not averse to finding philosophical meaning or justification for the schema they employed, and they appealed to principles of the organization of knowledge, but the final goal was always a practical one. Classification pioneers such as Dewey and Cutter were generalists who were not primarily concerned with any particular subject area. To the extent that their interests were focused on particular fields, these were found in humanistic and social subjects, not in the natural sciences.

Among all of the major systems of book classification, the Library of Congress scheme (LC) was the one that achieved dominance in university and research libraries. In 1870 the US Copyright Office was by legislation placed in the Library of Congress, and the Library received copies of all publications submitted for copyright. The holdings of the Library increased and became more complete than any elsewhere, including the collections of major university libraries and large public libraries. This was particularly true for the small-edition research books and monographs in academic fields. In 1901 the Library established its catalogue card distribution service, which allowed libraries throughout America to receive catalogue cards for all published books. Erdlund (1976, 398), an historian of the Library, comments on the significance of this development: "By including the card distribution service in its functions, the Library, at that time a reference library to Congress with a small constituency consisting almost exclusively of congressmen and their staff members, was adopting a potentially enormous constituency—that of the total American library community." The importance of the LC system in the world of classification was apparent in the years following its establishment. While the major university libraries with their specialized collections containing

[1]LaMontagne (1961, 208) observes that Charles Cutter "foresaw the continuing growth of the library and knew that each change in the shelving of books entailed the changing of 'shelf marks'—a long and expensive process. Cutter therefore decided to abandon fixed location 'and to adopt a method which would allow books to be moved without changing the marks on the catalogues.'" The part in quotation marks is from (Cutter 1882, 6). Cutter's "bench marks" are what we refer to today as call numbers.

many older and foreign-language books continued to maintain a patchwork of local classification systems, the LC has made steady headway up to the present as the dominant and most widely used system of library classification.

The work of the library cataloguers in the decades around 1900 was carried out against the background of a broad nineteenth-century interest in the classification of knowledge. In the next four sections we examine how mathematical subjects were classified, from the most general level down to the specific level of particular subject areas in analysis.

2 Place of Mathematics in Classification Schemes

Prior to, and concurrent with, the development of library classification systems in the nineteenth century there was a great deal of interest in the general problem of the classification of knowledge.[2] Although this problem was a venerable one, going back to the Greeks, it was of special concern in the nineteenth century and was the object of extensive research and discussion of a kind particular to this period. Insofar as the sciences were concerned mathematics occupied a privileged place in this philosophical project. In his recent book "Why Is There Philosophy of Mathematics at All?" Hacking (2014) explores the prominent place that mathematics has played throughout history in the writings of the great philosophers. In a review of this book Siegel (2016, 253) observes that "mathematics has perennially fascinated philosophers, to the point that the philosophy of mathematics is not the philosophy of a special science—like the philosophy of physics or biology—but rather a central field of analytic philosophy." In all of the major classification schemes, mathematics is either placed first in a category of general or fundamental science—along with philosophy; or it is put at the beginning of the natural sciences, followed by subjects that follow a presumed reductionist foundational order that exists among them: physics, then chemistry and finally biology.

Although the majority of the thinkers at the end of the nineteenth century interested in the organization of knowledge were neither scientists nor mathematicians, they possessed definite ideas about the classification of scientific subjects. A recurring theme was the coupling of mathematics with philosophy and its separation from subjects in natural science such as physics, astronomy, and chemistry. This point of view seems to have reflected the general and fairly widespread influence of philosophical logicism on contemporary scientific thought.

The Harvard psychologist Hugo Münsterberg was responsible for the scientific plan of The International Congress of Arts and Science held in 1904 at the St. Louis Exposition. Knowledge was divided into seven divisions, and each division was composed of a collection of departments. The divisions were: (A) normative sciences; (B) historical sciences; (C) physical science; (D) mental science; (E) utilitarian science; (F) social regulation; and (G) social culture (Bliss 1929, 375–380).

[2]For a general survey of work on classification in the nineteenth century see Dolby (1979).

Philosophy and mathematics comprised division A, while physics and the other traditional natural sciences made up division C.

The philosophical view of the place of mathematics within knowledge never really found favor with librarians who worked on the concrete project of book classification. However the affinity of mathematics with logic—if not philosophy—and its separation from the natural sciences was a prominent feature of Brown's 1906 subject classification scheme. Brown held that the classification of books should reflect the classification of knowledge, so that library classification was never a purely contingent project of information retrieval. Brown asserted that logic and mathematics should be grouped together under "Generalia" and should precede all other branches of knowledge, being preliminary to any field of investigation, from physics to economics to philosophy and history, or anything else. Classes of knowledge were given by Brown in the following order: generalia, physical science, biological science, medical science, agriculture and domestic arts, philosophy and religion, social and political science, language and literature, literary forms, and history and geography.

Brown's classification was also distinctive in its positioning of practical subjects adjacent to their presumed theoretical counterparts. Following the section on mathematics there would be books on painting and sculpture. There were two reasons for such an arrangement. First, fine and graphic art exemplified the visual point of view of geometry and could be regarded in some sense as the embodiment of geometric ideas. Second, Brown believed that the subject of visual representation was fundamental in character and that familiarity with it was necessary for its use in various applied fields of investigation and work, hence its placement under generalia. (It may seem odd that the section of the library stacks devoted to mathematics would be followed by books on Flemish art, while books on physics would be located a few aisles over—but such was Brown's rather idiosyncratic notion of book classification.)

A librarian influenced by Brown was Henry Bliss of the City College of New York. More than Brown, Bliss emphasized the affinity of mathematics with philosophy and logic and its separation from science. He wrote two books on the organization of knowledge: the first (1929) was a general and somewhat philosophical work, while the second (1933) was directed more specifically to the classification of books. Bliss regarded mathematics more as a method than a branch of science, and he believed that grounding in logic was proper preparation for its study. He observed (1929, 258) that "Logic is usually regarded as a branch of philosophy and the close relation of philosophical thought to mathematical thought is often affirmed." He had many criticisms of the Library of Congress classification scheme, including the position in LC of mathematics: "In a broader aspect the separation of Sciences in Q from philosophy in B involves such an unscientific and unphilosophic consequence as separating Philosophy of Science from Philosophy of Knowledge, and more generally separating Logic (BC) from Mathematics (QA), despite the claims of both logicians and mathematicians that their studies are inseparable."

The conception of Münsterberg and Bliss was implicitly rejected by thinkers concerned only with the organization and classification of subjects in the natural sciences. For them, mathematics clearly had to be included as a subject area, usually at the beginning of the classification, while philosophy did not appear at all. Any viable classification scheme would need to reflect the place of subjects in the real world. Classifying mathematics with philosophy and separating it from physics and engineering might be sensible in the domain of humanistic thought, but it made little sense in actual practice.

In the Cutter, Dewey and LC classification schemes mathematics is separated from philosophy and grouped with the natural sciences. In the Dewey system, philosophy is placed near the beginning under 100 and is followed by theology (200), sociology (300), and philology (400). The natural sciences comprise the 500s, with mathematics (510) as the first science subject proper, followed by astronomy (520), physics (530), and chemistry (540).

Cutter also grouped philosophy near the beginning under the letter B, where it was followed by religion and theology (C and D), biography (E), history and geography (F), social sciences (H), and natural sciences and applications (L). The presentation of subjects under L followed the order mathematics, physics, chemistry, and astronomy, with designations via subscripts: mathematics (L_B), physics (L_H), chemistry (L_O) and astronomy (L_R). Cutter classified the remaining science categories under the letters M through S: natural history as well as geology and biology (M); botany (N), zoology (O), anthropology and ethnology (P), and medicine (Q).

The LC system seems to have been patterned after Cutter (1891–93) and the placement of philosophy with respect to the natural sciences follows this earlier system. Sayers (1915–1916, 135) observes that "The outline of the [LC] classification is almost directly based upon The Expansive system, as a comparative paradigm of the two will demonstrate." The LC will be the subject of more detailed study in sections 4 and 5.

3 Scope of Mathematics in Classification Schemes

Until the nineteenth-century mathematics was interpreted broadly to include subjects that today would be regarded as part of astronomy, physics, or engineering. But by the second half of the nineteenth century, when library classification systems were being developed, the scope of mathematics had narrowed substantially.

Writers such as Münsterberg and Bliss who viewed logic and mathematics as kindred subjects and grouped mathematics with philosophy adhered to a conception of mathematics that certainly did not include subjects in physics such as mechanics. However, these thinkers did not represent scientists and mathematicians themselves. Among the latter mathematics was a subject that involved traditional logic only very peripherally if at all. Discussions of the scope and relative position of purely scientific subjects in the nineteenth century focused on what was called the hierarchy of sciences, a notion introduced by Comte in 1830 in the second lesson of his *Cours de philosophie positive*. Comte believed that there is a natural progression of scientific subjects, beginning with mathematics, passing through astronomy, physics, chemistry, and biology, and ending with sociology. This hierarchy could be justified on methodological or philosophical grounds, and was often taken for granted for practical reasons. The Comtean hierarchy of sciences was accepted by virtually all of the systems of book classification, and survives to the present.

Among those writers who were primarily interested in the natural sciences, mathematics was placed within science, at the very beginning of the Comtean hierarchy. An important figure was the French physicist André-Marie Ampère who along with Comte and some other French figures of the period was a mathematical empiricist in orientation. These authors separated mathematics completely from philosophy, which tended to occupy a lower position in the overall scheme of knowledge and learning than it had traditionally held. Mechanics was a kind of hybrid subject, part of mathematics and different from subjects in physics, but distinct from arithmetic and geometry in possessing a physical character.

Ampère (1834) presented a rather detailed and elaborate classification scheme for the sciences in his *Essai sur la philosophie des sciences* of 1834. The mathematical sciences were made up of arithmetic, geometry, mechanics, and astronomy. Arithmetic and geometry were mathematical subjects "proprement dits," while mechanics and astronomy were physico-mathematical in character. The physical sciences included atomic theory and chemistry.

Ampère's point of view was reflected in some later French writers on scientific classification. Thus Renouvier (1859) in his *Essais de critique générale* put rational mechanics and applied mathematics together with mathematical subjects (arithmetic, algebra, mathematical analysis, geometry) in the category of Logical Sciences, which were to be distinguished from Physical Sciences, the latter including astronomy. At the end of the century Goblot's *Essai sur la classification des sciences* (1898) positioned mechanics as part of mathematics and distinguished it from physics. The estimable Scottish authority Flint (1904, 278), in his survey of work on classification, seemed to regard mechanics as part of mathematics, writing that mechanics "is as abstract as Geometry, and in its applications is not more concrete," and "Mechanics is both abstract and concrete, both quantitative and qualitative, and cannot be denied to be on the borderland between mathematical and physical science."[3]

Although Ampère's understanding of the scope of mathematics was adopted by some later authors, the view that came to be much more common as the century progressed was just the opposite. There was a decided shift away from the view that mechanics was part of mathematics. In the last part of the century both humanistic and scientific thinkers interpreted mathematics as a subject more or less co-extensive with what today would be called pure mathematics.

[3] Flint (1904, 222–223 and 308–312) gives accounts of the classifications of Renouvier and Goblot.

This shift is apparent in the writings of the English polymath Herbert Spencer (1864), who published in 1864 *The Classification of the Sciences*. Spencer opposed Comte's hierarchy, mainly on the grounds of the reductionist ordering of the sciences along a linear sequence that it implied. Spencer was among that group of thinkers who believed that logic and mathematics were closely connected and distinguished by their abstractness from the natural sciences. Mathematics and logic dealt with relations, while the natural sciences dealt with objects. Rather than putting the natural sciences into a sequence he divided them into two distinct groups: the abstract-concrete sciences, consisting of mechanics, physics and chemistry; and the concrete sciences, consisting of astronomy, geology, biology, psychology, and sociology.

The exclusion of mechanics from mathematics was also advocated by the Austrian physicist Ernst Mach, who published his noted critical and historical account of mechanics in 1883. Although Mach's philosophy shared similarities with the empiricist outlook of Comte and Ampère, he insisted that mechanics was not part of mathematics. At the beginning of the preface to his book he proclaimed:

> Mechanics will here be treated, not as a branch of mathematics, but as one of the physical sciences. If the reader's interest is in that side of the subject, if he is curious to know how the principles of mechanics have been ascertained, from what sources they take their origin, and how far they can be regarded as permanent acquisitions, he will find, I hope, in these pages some enlightenment. All this, the positive and physical essence of mechanics, which makes its chief and highest interest for a student of nature, is in existing treatises completely buried and concealed beneath a mass of technical considerations. (Mach 1883, i)[4]

Mach was influenced by his phenomenological understanding of mechanics and his belief that *a priori* metaphysical conceptions had no place in physics, a mistake that could arise if mechanics was taken as part of mathematics. There were also important developments in nineteenth-century mathematics that influenced scientific thought in the century's second half. In a footnote toward the end of his book Mach discussed the discovery of non-Euclidean geometry. This discovery showed that geometry was not simply a description of spatial reality, for there were multiple geometries and only one spatial reality. Mathematics including geometry was evidently about intellectual structures, while mechanics was about objects in the external world. Non-Euclidean geometries existed, but non-inertial physics did not. Mach was opposed to the interpretation of the properties of real space ("die Eigenschaften des gegebenen Raumes") by what he called "the pseudo-theories of geometry that seek to excogitate these properties by metaphysical arguments."

The common view among the classifiers of science in the second half of the century was that mathematics did not include mechanics. This fact is apparent in a broad range of authors discussed by Flint in his 1904 historical survey. Whewell in 1858 distinguished mathematics (arithmetic, geometry, algebra, differentials) from

[4]English translation is by Thomas J. McCormack from the 1893 English edition of Mach's book, *The Science of Mechanics; A Critical and Historical Account of Its Development* (Open Court, Chicago).

astronomy and mechanics (Flint 1904, 198). Wilson in 1856 separated mechanics, which he called a practical science, from mathematical subjects (arithmetic, algebra, geometry, calculus) which made up, with the study of method and ontology, the pure sciences (Flint 1904, 215–216). de Roberty in an 1881 book on sociology separated mathematics from mechanics, regarding the latter as a descriptive science (Flint 1904, 263–4). In his 1887 book *Versuch einer concreten Logik*, the Prague philosopher Tomáš Masaryk (1887) advocated a hierarchal conception of science, placing mathematics first and assigning mechanics to a second group (Flint 1904, 277–8).[5] Masaryk followed Mach in explicitly separating mechanics from mathematics. In 1870 the Scottish philosopher Alexander Bain asserted that mathematics was distinct from mechanics, and placed the latter with physics (Flint 1904, 241–2). In Pearson's *Grammar of Science* of 1892 logic and mathematics were classified as abstract sciences, while mechanics was one of the concrete sciences. One year later Raoul de la Grasserie (1893) followed Herbert Spencer in classifying mathematics as an abstract science and mechanics as abstract-concrete (Flint 1904, 289–292). Writing in the early 1930s but expressing long-held views, Bliss (1933, 293) asserted that the possibility of a mathematical treatment of mechanics "should not mislead scientists to admit the claims of some mathematicians that Mechanics is merely a branch of Mathematics. That is not true even of Rational, or Analytic Mechanics, which of course should not be dissevered from the sub-science as a whole."

The book-classification schemes at the end of the century were united in limiting the scope of mathematics, and in either placing mechanics within physics or including it as a subject area in its own right. In Cutter, mechanics was put with physics rather than mathematics, while astronomy was made a distinct subject area, after chemistry. Although Dewey (1886) had included some applied subjects in mathematics, mechanics was placed in physics, along with optics, thermodynamics, and electromagnetism. In the International Catalogue of Scientific Literature (see §4) mechanics received its own subject area, intermediate between mathematics and physics. In the systems of both Brown and Bliss, mechanics is separated from mathematics and classified as a physics subject, along with thermodynamics and electromagnetism.

Alone among the major classification systems, the LC scheme placed mechanics under mathematics, and situated astronomy as a subject field between mathematics and physics. The title of the original LC volume on mathematics is worded "Class QA: Mathematics (Including Analytic Mechanics)." It is not entirely clear why the architects of LC proceeded this way, but the grouping of mechanics within mathematics is a singular feature of the LC classification system that continues to the present.

[5]Flint (1904, 277) mistakenly gives the date of publication of Masaryk's book as 1866. Masaryk was born in 1850 and entered the University of Vienna in 1872.

4 The Place of Calculus/Analysis in Classification Schemes for Mathematics

Comte's distinction between abstract mathematics consisting of arithmetic, algebra and calculus, on the one hand, and concrete mathematics, consisting of geometry and mechanics on the other, reflected a classificatory order that placed calculus ahead of geometry. It was also in keeping with the prevailing conception in French mathematics of calculus as a form of "algebraic analysis," the very title of Augustin Cauchy's famous textbook of 1821 on the calculus.

In his 1834 book Ampère introduced neologisms to designate the various subject areas of mathematics. What he called "arithmologie" was divided into two parts, the first consisting of arithmetic and algebra, and the second consisting of the theory of functions and the theory of probabilities. The theory of functions encompassed calculus-related parts of mathematics. Geometry was the second subject area of mathematics, under which Ampère placed synthetic and analytic geometry, as well as the theory of lines and surfaces and something called molecular geometry. The last subject area of mathematics consisted of the physico-mathematical subjects mechanics and astronomy (the latter called "Urinologie" by Ampère.) Mechanics in turn was divided into elementary and transcendental mechanics, while astronomy was divided into general astronomy and celestial mechanics.

Among the many writers who wrote on classification of science from the 1840s to the end of the century, the predominant tendency was to depart from Comte and Ampère by placing geometry ahead of calculus. Mathematical subjects were placed in the standard order: arithmetic, algebra, geometry, and calculus. Whewell in 1840 conceived of mathematics as the subjects "Geometry, Arithmetic, Algebra, and Differentials, and based on the ideas of space, time, number, sign, and limit" (Flint 1904, 199). Bain in 1870 divided mathematics into arithmetic, algebra, geometry, algebraic geometry, and the higher calculus (the latter dealing with incommensurable magnitudes) (Flint 1904, 199). Wilson in 1856 gave the order arithmetic, geometry, algebra, calculus, trigonometry, and analytic geometry (Flint 1904, 216). Janet in 1897 used abstraction as something that distinguished arithmetic, geometry and mechanics from algebra and the differential and integral calculus (Flint 1904, 304). Flint (1904, p. 278) himself wrote that "Arithmetic and Geometry are very different both as to matter and method from Calculus and Kinematics."[6]

[6]An exception to the prevailing consensus was Karl Pearson, who in his *Grammar of Science* (1892) put theory of functions and calculus together with arithmetic and algebra, these subjects dealing with quantity, while geometry was classified as a distinct subject area dealing with space (Flint p. 296). Earlier the Paris book seller Jacques-Charles Brunet (1814, 1860) in his pioneering classification scheme placed mathematical subjects in the order arithmetic, algebra, calculus, and geometry. Brunet was presumably influenced by Comte and Ampère. Brunet's catalogue was exceptional among all classification schemes in placing mathematics at the end of the sciences, following philosophy, physics, chemistry, geology, biology, and medicine.

With the exception of the Library of Congress, the major library classification schemes around 1900 placed geometry before calculus. Dewey and Cutter both adopted the order arithmetic, algebra, geometry, trigonometry, and calculus, while Brown presented these subjects in the order arithmetic, algebra, geometry, calculus, and trigonometry. The librarians presumably were guided by historical and pedagogical considerations: calculus had originated as a set of methods for the study of curves and surfaces, and calculus was a more advanced teaching subject than elementary geometry and therefore was placed after it. The librarians may also have perceived the natural order to be one of successive abstraction, and calculus and higher analysis were viewed as more abstract than geometry.

Although the subject of the present essay is the classification of books, it is necessary to look at how periodical mathematical literature was classified by subject in the second half of the nineteenth century, as this would bear directly on the classification scheme for mathematical books adopted by the LC. Unlike book classification, which was aimed at a very broad readership at various levels of engagement with the subject, the practices followed by journals reflected the outlook of advanced researchers in the field. Insofar as the ordering of subjects is concerned, the point of view was essentially a continuation of the French outlook expressed by Comte and Ampère early in the century. The *Zeitschrift für Mathematik und Physik*, founded in 1856, was one of the first journals to explicitly divide its contents into subject categories. The latter were presented in the order arithmetic and analysis, geometry, mechanics, optics, electricity and Galvinism, and smaller and miscellaneous subjects. The grouping of analysis with arithmetic and its placement ahead of geometry reflected the prevailing view of advanced researchers, and indicated more generally the well-known "arithmetization of analysis" in the development of mathematics in the nineteenth century. Calculus in its original formulation was known as "fine geometry," and such eighteenth-century masters of analysis as Euler and Lagrange were known as geometers. By the second half of the nineteenth century the research picture had shifted radically, and geometry had become something of a subsidiary subject with respect to the primary grounding of advanced mathematics in arithmetic and analysis.

Carl Ohrtmann and Felix Müller were Berlin gymnasium teachers of mathematics who founded in 1871 the abstracting periodical *Jahrbuch über die Fortschritte der Mathematik*. There was a large increase in the growth of mathematical literature in the nineteenth century, and a corresponding need to assist researchers in navigating materials published in their fields. Ohrtmann and Müller modelled the *Jahrbuch* after an abstracting journal for physics that had already been in existence for close to twenty-five years, the *Fortschritte der Physik/Physikalische Berichte*. Although the publications reviewed in the *Jahrbuch* consisted mainly of periodical literature, books were also included. Subjects were presented in the order history and philosophy, algebra, number theory, series, differential and integral calculus, function theory (complex functions), pure, elementary and synthetic geometry, analytic geometry, mechanics, mathematical physics (electromagnetism, theory of heat, optics), and geodesy and astronomy.[7] Since there was already a physics abstracting journal, the physics subjects included in the *Jahrbuch* were ones in which the treatment was very mathematical.

At the end of the century the Royal Society of London established the International Catalogue of Scientific Literature A Mathematics (1902), a major international bibliographic project that was intended to cover both periodical and book literature. In this work mathematics (which was also referred to as "pure mathematics") was divided into the following subject areas: fundamental concepts, algebra and number theory, analysis, and geometry. This ordering of subjects became canonical in the classification of twentieth-century mathematical literature, at least as this was followed by the LC and mathematical abstracting services. (It should be noted that the Dewey system continued to place geometry before calculus and analysis up until the late 1960s, at which time its schedules were revised and brought into alignment with the LC.)

The classification schedules for mathematical subjects in the original LC system of 1905 were compiled by J. David Thompson, chief of the science section, under the direction of Martel, head of classification for the whole of LC (See Library of Congress Classification Class Q Science (1905).). Thompson was a native of England who had studied mathematics at the University of Cambridge, graduating 16th Wrangler in 1895. In the preface to the volume on science he (1905, 3) states that he has relied notably on the schedules of the International Catalogue of Scientific Literature. While the overall scheme of the LC system was patterned on the Cutter system of classification, the organization of scientific subjects followed the ICSL. Insofar as advanced mathematical subject areas were concerned, Thompson followed the ICSL very closely. The 1905 edition of the LC science schedules was republished in multiple later editions, each containing modifications and extensions of the original scheme.

In the 1930s there were two new library classification systems, Bliss's bibliographic classification and Ranganathan's (1933) colon classification. Although Bliss presented the three subject areas of mathematics as arithmetic-and-algebra, geometry and analysis, for the purposes of classification he placed them in the order arithmetic-and-algebra, analysis, and geometry. He made this change for reasons of what he called "collocation," apparently referring to the usage established by the ICSL and the LC. Ranganathan also classified mathematics subjects in the order arithmetic, algebra, analysis, and geometry, and followed LC in including mechanics within mathematics. In a departure from all other classification schemes he placed astronomy within mathematics.

[7] Göbel (2008, 9–13) observes that "Später bildeten diese Abschnitte die Grundlage für den Aufbau einer Mathematik-Klassifikation." ("These divisions later formed the basis for the construction of a mathematics classification.")

5 Analysis in the LC Classification for Mathematics: The Case of Functions of a Complex Variable

In the LC classification system books on science are classified under Q, and those on mathematics are classified under QA. In 1905 some parts of mathematics hardly existed yet as recognized subject areas. In the ICSL under arithmetic there was a subject entry on “aggregates,” what would later be called the theory of sets, but there was no entry at all for this subject in the LC. When Abraham Fraenkel’s *Einleitung in die Mengenlehre* appeared in 1919 it was classified in the LC under foundations of arithmetic (QA248) in the algebra section, and that became the standard LC subject classification for books on set theory.

A part of mathematics that was very well established in 1905 was analysis, and books on this subject received call numbers in the range from QA300 to QA400. The theory of functions was designated QA331 and was made up of books we would regard today as belonging to complex analysis. The theory of functions of a real variable came to be designated QA331.5, being regarded as a branch or offshoot of the theory of functions. The classification scheme is evident in the following two books on analysis from the early years of the century:

QA331 Heinrich Burkhard, *Theory of Functions of a Complex Variable* (1913)
QA331.5 James Pierpont, *Lectures on the Theory of Functions of Real Variables* (1905–12)

When Lars Ahlfhors’ *Complex Analysis* was published in 1953 it was given the LC subject designation QA331. In the 1960s complex analysis replaced the theory of functions as the standard subject name for the theory of functions of a complex variable (Figure 1).[8] At this time one also began to see the publication of books

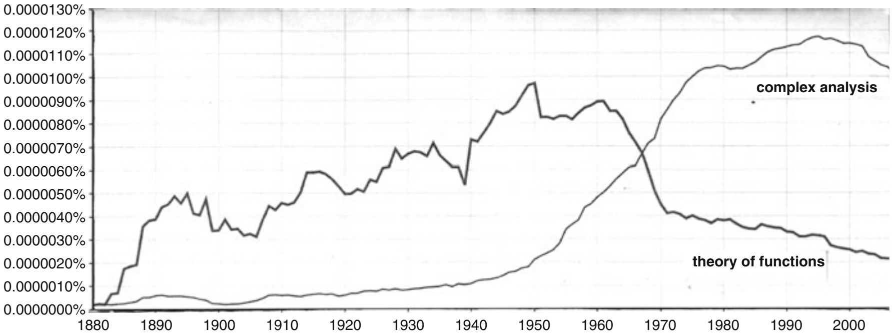

Fig. 1 Google Ngram: theory of functions, complex analysis

[8]An interesting graphical illustration of this change in usage is provided by the Google Ngram (Michel et al. 2011) for the frequency of the terms “theory of functions” and “complex analysis” for the period from 1880 to 2008. See Figure 1.

with the term "real analysis" in the title. H.L. Royden's *Real Analysis* appeared in 1963 and was given the subject designation QA331.5. Thus real analysis was envisaged in this classification scheme as an offshoot of complex analysis. The earlier subject classifications QA331 (theory of functions, implicitly functions of a complex variable) and QA331.5 (theory of functions of a real variable) mapped onto the new subject names complex analysis (QA331) and real analysis (QA331.5).

In the LC books on analysis with the classification QA300 are devoted to the more general parts of analysis and the foundations of the subject. A widely used primer on analysis for senior undergraduate and graduate students from the 1950s and 1960s was Walter Rudin's *Principles of Analysis* (1953 and later editions). Rudin's book was classified under QA300. We have the classification sequence:

Q Science
  QA Mathematics
    QA300 Rudin *Principles of Analysis*
      QA331 Ahlfors *Complex Analysis*
        QA331.5 Royden *Real Analysis*

By the 1970s some books on real analysis were assigned the designation QA300, and thus were understood to belong to more general parts of analysis, prior in the classification scheme to complex analysis. Other books on real analysis continued to receive the traditional designation QA331.5. There was an overhaul of LC mathematical analysis subject designations in the 1980s, and this change is contained in the current schedule that may be found online (http://www.loc.gov/aba/publications/FreeLCC/Q-text.pdf). The change had been made by around 1990. Here is how the breakdown for subjects in analysis is now given:

Analysis
QA300 General works, treatises, and textbooks

Theory of functions
QA 331 General works, treatises, and advanced textbooks
QA 331.3 Elementary textbooks
QA 331.5 Functions of real variables
QA 331.7 Functions of complex variables
  Riemann surfaces including multiform, uniform functions

Evidently the QA331 section dealing with the theory of functions has been reorganized to reflect the standard order of subject presentation: first general works, followed by elementary presentations, and then according to some presumably natural principle, an ordered list of the subject areas that fall under the theory of functions.

An old principle of book classification followed by the LC that is very useful to the historian is that books are not reclassified when a revision, either major or minor, of the classification system takes place.[9] This seems to be partly for practical reasons—it would be difficult for libraries to be continually reclassifying the materials in their collections. It should be noted that although the classification of a book is not changed, in the LC a later edition of a given book may have a different call number. For example, Stanley G. Krantz's *Function Theory of Several Complex Variables* was classified as QA331 when it appeared in 1982, a designation that remains unchanged to this day, while the second edition of this book in 1992 received the call number QA331.7.

The principle of the organization of scientific subjects which is followed in library catalogues is foundational: more basic subjects come first, followed by progressively more complex subjects. Underlying the conception of foundation is a building metaphor, invoking the structure and construction of a building. A classification where real analysis is placed before complex analysis is consistent with a foundational conception of subject classification. In the original LC classification, where the theory of functions of a real variable is a sub-subject of the theory of functions, David Thompson was presumably thinking of classification in a somewhat different way, as a division in which the complete subject comes first, and where one proceeds from there to obtain various special subject areas that fall within the general subject. In certain contexts this second approach to classification may seem more natural or practical, as it would, for example, if one were organizing goods in a department store. To find a given make of coffee maker one would locate the section on household goods, proceed to the section on kitchen supplies, and then find the section on kitchen appliances, ending finally in the section on coffee makers. The conventional ordering of intellectual subjects—at least ones in science—follows a different, foundational principle that is inherent in the epistemological character of the subject matter.

It should be noted that there was also an evolution in the classifications schedules for mathematics employed in the Dewey Decimal system. The original Dewey schedule from 1885 for books on mathematics was:

511 Arithmetic
512 Algebra
513–516 Geometry
517 Calculus (analysis)

[9]While there is a general conservatism among librarians with respect to classification, in the case of the Dewey Decimal system there have been revisions of the classification that have been retroactively applied by some libraries to books in their collections. For example, before 1970 books on analysis were classified by Dewey under 517 (after geometry), whereas after 1970 such books were assigned the classification 515 (before geometry). In the public libraries of Cleveland and Cincinnati the older books retain their original classifications. However, in the library of the University of Illinois at Champaign-Urbana, where the Dewey system is used, the older books on analysis have been assigned the new classification numbers. This is also generally true of the Toronto Public Library.

Around 1970 the Dewey classification for books on analysis was changed from 517 to 515, while books in geometry that originally would have been classified under 515 were classified as 516. For example, a book on complex analysis published in 1965 was classified under 517, while a book on the same subject by the 1970s was classified under 515. In addition, the classifications for arithmetic and algebra were reversed, so that arithmetic became 513 while algebra remained at 512. The current Dewey classification schedule for books in mathematics is:

512 Algebra
513 Arithmetic
514 Topology
515 Analysis
516 Geometry

The change in the classification brought the Dewey system into conformity with the LC as well as with the classifications used by abstracting periodicals such as *Mathematical Reviews*.

Prior to about 1970 in the Dewey system books on functions of a complex variable and books on functions of a real variable where both standardly catalogued under the call number 517.5. There was no relative placement of one group with respect to the other, as there was in LC. When books with complex analysis and real analysis in their titles appeared in the 1950s and early 1960s they were all given the same call number 517.5. During the later 1960s books on complex analysis were being classified as 517.8, while ones on real analysis continued to receive 517.5. After 1969 or 1970 all books on analysis shifted from 517 to 515, with books on real analysis being assigned 515.8, and ones on complex analysis being assigned 515.9.[10]

Acknowledgements I would like to thank Hardy Grant for advice on an earlier draft of this paper and Matthew Edwards for helpful discussions. I am grateful to two referees for their comments on the paper. I am also grateful to Noah Stemeroff for help with LaTeX and for his careful proof reading of the paper.

References

Ampère, André-Marie. (1834) *Essai sur la philosophie des sciences, ou exposition analytique d'une classification naturelle de toutes les connaissances humaines. Tome 1*. Mallet-Bachelier, Paris.

Bain, Alexander. (1870) *Logic Part First Deduction*. Longmans, Green, Reader and Dyer, London.

Bliss, Henry E. (1929) *The Organization of Knowledge and the System of the Sciences*. Henry Holt and Company, New York.

[10]There was not complete consistency in cataloguing during the transition years between about 1968 and 1972. It should also be noted that books on real analysis were sometimes assigned 517 rather than 517.5 (before 1970) and 515 rather than 515.8 (after 1970).

Bliss, Henry E. (1933) *The Organization of Knowledge in Libraries and the Subject-Approach to Books*, H.W. Wilson Company, New York.

Bliss, Henry E. (1935) *A System of Bibliographic Classification*. The H.W. Wilson Company, New York.

Brown, James Duff. (1906) *Subject Classification, with tables, indexes, etc. for the sub-division of subjects*. Library Supply, London.

Brunet, Jacques-Charles. (1814, 1860) *Manuel du libraire et de l'amateur de livres*. Dido Frères, Paris. The first edition of this work was published in 1814. An English translation and introduction by Donald Bruce McKeon of the 1864 edition appeared in 1976 as No. 1 of the Louisiana State University Graduate School of Library Science Occasional Papers. Baton Rouge. In an appendix the translator gives a statistical breakdown of the contents of Brunet's bookshop, using the total number of bench marks or call numbers each subject receives as a percentage of the total number of bench marks overall. Arts and Sciences constituted 22.7% of the contents of the collection, while the mathematical sciences constituted 15.2% of Arts and Sciences. From this one may deduce that the mathematical sciences represented 3.5% of the overall collection.

Brunet, Jacques-Charles. (1860, 1864) *Manuel du libraire de l'amateur de livres. Tome premier*. Librairie de Firmin Didot Frères, fils et Cie. Five more volumes appeared up to 1865.

Comte, Auguste. (1830) *Cours de philosophie positive tome premier Contenant les préliminaires généraux et la philosophie mathématique*. Rouen Frères, Paris.

Cutter, Charles A. Boston (1882) *Athenaeum; How to Get Books, With an Explanation of the New Way of Marking Books*. Rockwell Churchill, Boston.

Cutter, Charles A. (1891–93) *Expansive Classification Part I: The First Six Classifications*. Boston.

de la Grasserie, Raoul. (1893) *De la classification objective et subjective des arts, de la littérature des sciences*. F. Alcan, Paris.

de Roberty, Eugène. (1881) *La Sociologie*. Germer Baillière, Paris.

Dewey, Melvil. (1886) *A Classification and Subject Index for Cataloguing and Arranging the Books and Pamphlets of a Library*. Case, Lockwood & Brainard Company, Amherst, Mass.

Dolby, R.G.A. (1979) "Classification of the Sciences The Nineteenth Century Tradition," in (Roy Ellen and David Reason, 1979), pp. 167–193.

Ellen, Roy F. and David Reason. (1979) *Classifications in their Social Context*. Academic Press, London, New York and San Francisco.

Erdlund, Paul.(1976) "A Monster and a Miracle: The Cataloging Distribution Service of the Library of Congress, 1901–1976," *The Quarterly Journal of the Library of Congress* 33, pp. 383–421.

Flint, Robert. (1904) *Philosophy as Scientia Scientiarum and History of Classifications of the Sciences*. William Blackwood and Sons, Edinburgh and London.

Göbel, Silke. (2008) "Die Anfänge des Wissensmanagements in der Mathematik: das „Jahrbuch über die Fortschritte der Mathematik"", in Bernd Wegner (Ed.), *Mathematik in Blickpunkt*, pp. 9–13. FIZ Karlsruhe.

Goblot, Edmond. (1898) *Essai sur la classification des sciences*. Germer Baillière, Paris.

Hacking, Ian.(2014) *Why Is There Philosophy of Mathematics At All?* Cambridge University Press.

International Catalogue of Scientific Literature A Mathematics. (1902) Royal Society of London, London.

Janet, Paul. (1897) *Principes de métaphysique et de psychologie Leçons professées à la Faculté des Lettres de Paris 1888–1894 tome 1*. Librairie Ch. Delagrave, Paris.

LaMontagne, Leo E. (1961) *American Library Classification, with special reference to the Library of Congress*. Shoe String Press, Hamden, Conn.

Library of Congress Classification Class Q Science. (1905) Government Printing Office, Washington.

Mach, Ernst. (1883) *Die Mechanik in ihrer Entwickelung historisch-kritisch dargestellt*. Brockhaus, Leipzig.

Masaryk, Tomáš. (1887) *Versuch einer concreten Logik*. Verlag von Carl Konegen, Vienna.

Michel, Jean-Baptiste et al. (2011) "Quantitative Analysis of Culture Using Millions of Digitized Books," Science Vol. 331 (Issue 6014), pp. 176–182.

Pearson, Karl. (1892) *Grammar of Science*. Walter Scott, London.
Ranganathan, S.R. (1933) *Colon Classification*, Madras Library Association, Madras, E. Goldston, London.
Renouvier, Charles. (1859) *Essais de critique générale*. A. Colin, Paris.
Sayers, W.C. Berwick. (1915–1916) *Canons of Classification*. Grafton and Co., London and The H.W. Wilson Company, White Plains, N.Y.
Siegel, Max H. (2016) Book review of (Hacking, 2014). *Mind* 125 (497), pp. 252–6.
Spencer, Herbert . (1864) *The Classification of the Sciences*. D Appleton and Company, New York.
Whewell, William. (1858) *Novum Organon Renovatum: Being the Second Part of the Philosophy of the inductive Sciences*. J.W. Parker, London.
Wilson, William D. (1856) *Elementary Treatise of Logic*. D. Appleton and Company, New York.

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