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Galuzzi, Massimo (I-MILAN)

Lagrange's essay "Recherches sur la manière de former des tables des planètes d'après les seules observations". (English. English, French summary)

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Although Lagrange's 1775 paper was ostensibly devoted to the investigation of sequences of numbers arising in astronomical tables, its contribution to astronomical measurement was in fact very limited. The paper was however an interesting and important treatise on theoretical methods in numerical analysis. The article under review provides the most detailed account to date of Lagrange's paper. Writing from the standpoint of modern research on generating functions and recurrent sequences, the author provides a clear exposition of Lagrange's methods, using current notation and providing (in a modernized form) some of the details Lagrange omitted.

A sequence $d_0, d_1, d_2, \cdots$ is said to be recurrent if there exist constants $\lambda_1, \cdots, \lambda_n$ such that $d_i = \lambda_1 d_{i-1} + \lambda_2 d_{i-2} + \cdots + \lambda_n d_{i-n}$ for all $i \geq n$, where the first n terms are given. A generating function for a sequence $d_0, d_1, d_2, \cdots$ is a function whose Taylor expansion about zero is the power series $\sum_{n=0}^{\infty} d_n x^n$. Lagrange considered a particular class of trigonometric recurrent sequences that arise in astronomical tables. His goal was to find a generating function for this sequence of the form $P(x)/Q(x)$, where $P(x)$ and $Q(x)$ are polynomials and the degree of $P(x)$ is less than the degree of $Q(x)$. To accomplish this goal he developed an algorithm involving continued polynomial fractions. Galuzzi describes this algorithm and Lagrange's refinements of it. He also considers in some detail three numerical examples used by Lagrange to illustrate his method.

It should be noted that an explicit theory of generating functions was only first developed by Laplace in 1782. Nevertheless, Lagrange's 1775 paper contained important results in this direction. Indeed, Galuzzi writes that "the main contribution of Lagrange's paper (in my opinion) lies more in the direction of developing the theory of generating functions than in promoting numerical calculus".

Galuzzi makes reference in the article to several mathematical textbooks in the field of numerical anslysis that have been published in the last ten years. Although he is attentive to the motivation and conceptual milieu of Lagrange's original study, he is primarily interested in using the historical materials to explore and to deepen our understanding of the modern mathematical subject. He believes it is historically sensible to analyze Lagrange's methods from a modern perspective. He writes (p. 228): "But it is precisely the very ease with which we move from Lagrange's mathematics to modern combinatorics that makes manifest (in my opinion) the legitimacy of reading his mathematics as I do. And to present this point of view is obviously to exert a historical judgement."

Craig G. Fraser