

MR1121767 (92j:01022) 01A45 26-03 26A03

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★Images du continu. (French) [[Images of the continuum]]

*The Leibniz renaissance* (Firenze, 1986), 83–97, *Bibl. Storia Sci.*, 28, Olschki, Florence, 1989.

This article contains some observations on the way the continuum was understood in the 17th-century work of Galileo, Cavalieri, Viète, and Leibniz. The author notes that conceptions of the continuum emerged in the course of the development of geometrical theories; they did not precede these theories as one might logically expect.

Although Galileo argued in opposition to Aristotle that the continuum was composed of ultimate atoms or indivisibles, he did not believe that aggregates composed of such indivisibles could be quantitatively compared. Cavalieri, by contrast, made the assumption of this possibility the cornerstone of his investigation. By applying the theory of proportions to collections of indivisibles, he developed a theory of measure that did not require the method of exhaustion. Viète's analytic art helped to reconcile the conception of number within arithmetic algebra, which by the 16th century included surd numbers or radicals, with the classical conception of magnitude as continuous extension. The author observes: "Viète's image is that of an amphibious continuum, whose elements are simultaneously magnitudes and numbers."

Leibniz's notion of the continuum is shown to involve the property that in modern mathematics is called connectivity. A closed set  $X$  is said to be connected if it has the following property: If  $X$  is contained in the union of two closed sets  $Y$  and  $Z$ , and if  $X$  contains elements from both  $Y$  and  $Z$ , then there exist elements in the common intersection of  $X$ ,  $Y$  and  $Z$ . The essay concludes with a brief discussion of Leibniz's calculus. Leibniz believed that when a line is divided in two parts, two points of division are produced infinitesimally close together. This idea was related to his understanding of the tangent to a curve as a line joining two neighbouring points on the curve an infinitesimal distance apart.

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