Bulletin of the Canadian Society for the History and Philosophy of Mathematics **No. 74 May 2024, pp. 15-16**

R*ss*ll' from 1918;

- Jared M. Ifland, arguing that the Frege-Hilbert controversy was about more than ontology;
- Martin E. Flashman, suggesting that ideas for the philosophy of geometry may emerge out of attempts to teach the material to would-be teachers;
- Chanwoo Lee, examining various attempts at providing a foundation for mathematics that is explanatory and not just justificatory; and
- Owen Biesel, seeing whether sheaf theory might be relevant to coming up with beliefs that change over time or from one individual to another.

Attendance at some of the talks was about 75, and there was generally lively discussion.

Tom Drucker

Off the Shelf: Goldstine's *Calculus of Variations*

A History of the Calculus of Variations from the 17th through the 19th Century, by Herman H. Goldstine. New York: Springer-Verlag, 1980.

Herman Goldstine (1913–2004) pursued a career in the United States in computer engineering and science and is regarded as a prominent figure in the computer revolution. A permanent member of the Princeton Institute for Advanced Study (IAS), he was not an academic and did not teach students. His engagement with history consisted of three books he wrote in the 1970s, on computers, numerical analysis, and the calculus of variations.

The calculus of variations is a branch of analysis with a venerable history. Goldstine composed his book some forty years after he carried out doctoral research in the field as a young man at the University of Chicago. He was motivated by a belief that older histories were what he termed "hopelessly archaic." His approach was "to select those papers and authors whose works have played key roles in the classical calculus of variations as we understand the subject today" (Goldstine 1980, vii). The adjective "classical" referred to the subject that had taken form and was summarized in the writings of Chicago mathematicians Oskar Bolza and Gilbert Bliss. The most detailed statement was Bolza's 700-page tome of 1909, *Vorlesungen über Variationsrechnung* (published by B. G. Teubner), a greatly expanded edition of an English book Bolza wrote in 1904.

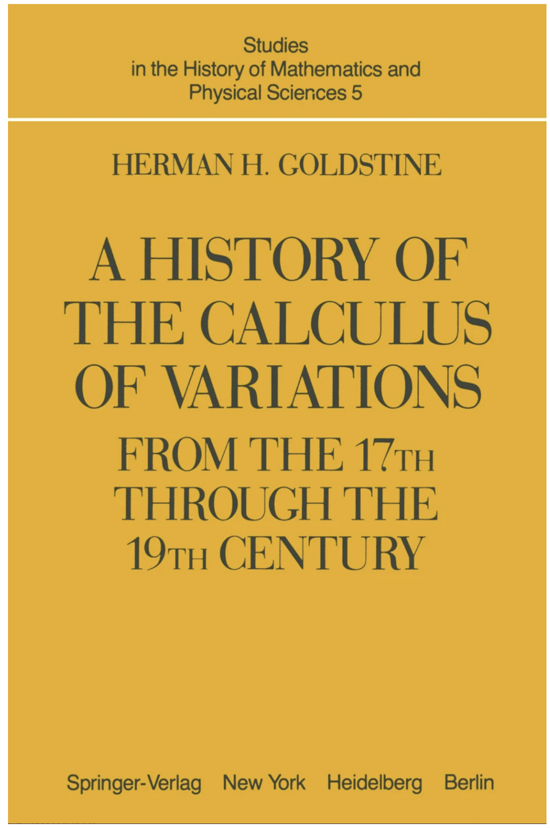
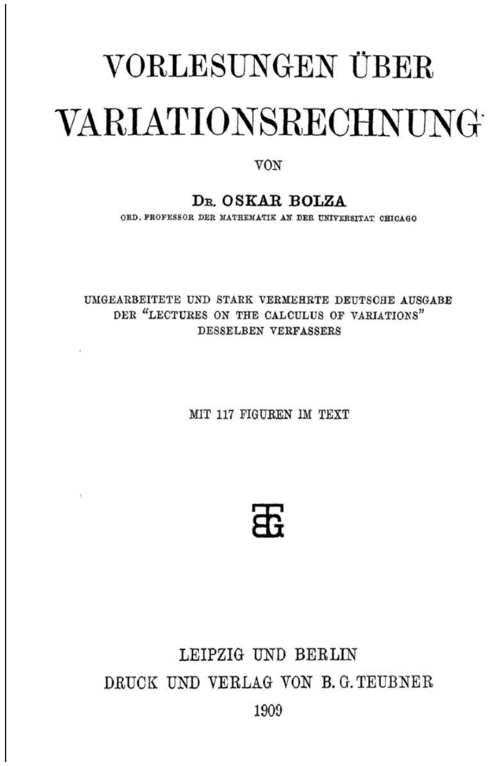


Figure 6: Goldstine's *History*.

Goldstine's book is presented in the form of a survey with particular emphasis on the period after 1800. He was adept at reading Latin, French, and German, and he wrote expository and generally readable accounts of original source material. He possessed an abiding and admirable interest in the mathematics itself. Astute insights into the historical nature of the mathematics were sometimes relegated to footnotes, although the main narrative also included occasional perceptive historical comments about the subject.

Although Goldstine wrote his book at the IAS he seems to have had little contact with historians of science at Princeton University. He did acknowledge discussions with Otto Neugebauer, but the latter's work on ancient and early modern exact science would have offered only limited historiographical guidance for the study of the development of modern mathematics. Goldstine's book on the history of the calculus of variations received critical reviews when it appeared. The assessment of historian of physics J. J. Cross was typical. Cross stated that the book was "a partial outline of the development of the pure mathematical aspects


VORLESUNGEN ÜBER

VARIATIONSRECHNUNG

VON

DR. OSKAR BOLZA

ORD. PROFESSOR DER MATHEMATIK AN DER UNIVERSITÄT CHICAGO

UMGEARBEITETE UND STARK VERMEHRTE DEUTSCHE AUSGABE
DER "LECTURES ON THE CALCULUS OF VARIATIONS"
DESSELBEN VERFASSERS

MIT 117 FIGUREN IM TEXT

LEIPZIG UND BERLIN
DRUCK UND VERLAG VON B. G. TEUBNER
1909

Figure 7: Bolza's 1909 book.

of this topic . . . since people, their backgrounds and their motivations are missing, as history this book is quite unsatisfactory in many respects" (*Annals of Science* 39 (1982): 519).

In defence of Goldstine, one might note that any historical study of past mathematics must be based on an exploration of the content of the subject. Still, even from the point of view of a mathematician looking at history there are some weaknesses in Goldstine's approach to the material. No notice is taken in his account of the range of nineteenth-century textbooks in the calculus of variations by authors in Europe and America. These writings are of interest in themselves and provide mathematical context to understand and assess discoveries in the research journals. Furthermore, the choice of noteworthy papers is subjective and, in some cases, does not lead to a proper mathematical or historical understanding of the work in question. An example is Jacobi's seminal 1837 article in Crelle's journal on the second variation, which was presented in a programmatic and incomplete form. As commentators of the period noted, Jacobi's discoveries came into focus when they were explicated in a doctoral memoir of 1841 by Charles Delaunay (not selected for consideration by Goldstine).

The last part of Goldstine's book is in fact of some historical interest itself as an indication of how the "classical" calculus of variations was understood by someone trained in this research tradition. The point of departure consisted of formulating a very general statement of the variational problem, something that was achieved by introducing constraints in the form of differential equations and applying a more abstract form of a multiplier rule. Within this setting, the main goal was investigating conditions that were sufficient to ensure a maximum or a minimum. This was established in one of two ways. The first was to examine the second variation, a project that was put in its modern form by Alfred Clebsch and Adolph Mayer at the middle of the century and continued around 1900 by Adolf Kneser and Gustav von Escherich. The second employed Weierstrassian field methods based on the fundamental concept of a field of extremals and was presented by researchers early in the twentieth century. Major figures here included Kneser, Ernst Zermelo, David Hilbert, and others. A prominent theme throughout work during this period involved questions of existence, which first arose in the 1860s and became a major concern by the new century.

It is necessary to also mention some novel developments that extend beyond the period covered in Goldstine's book but provide context for the mathematics he documented. There was a tendency to relate the subject to differential geometry which later in the century would give rise to calculus of variations in the large and Morse theory. New lines of research beyond what might be regarded as the classical subject occurred with the investigation of multiple integrals in the 1920s and 30s and the emergence of optimal control theory in the 1950s. There have been as well multiple lines of research examining the general problem of optimization. When *Mathematical Reviews* was founded in 1939, calculus of variations was an independent branch of analysis in the subject classification scheme. In today's *MSC* it is also grouped with optimal control and optimization, reflecting the post-classical evolution of the subject.

Craig Fraser

Quotations in Context

> "As for everything else, so for a mathematical theory: beauty can be perceived but not explained."