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★**A mathematical history of division in extreme and mean ratio.**

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The basis of this book, Chapters I and II, is a detailed study of the concept of division in extreme and mean ratio (DEMR) in Euclid's *Elements*. Chapters III–VI describe the pre-Euclidean development of DEMR, while Chapters VI and VII present the later Greek history. Non-Western contributions (the Arabic world, India and China) are examined in Chapter VIII, and a survey of the European history from the Middle Ages to 1800 is the subject of Chapter IX. In two appendices the author discusses the terminology throughout history for DEMR, and "traces the admiration, to put it mildly, that DEMR has evoked from mathematicians from Campanus to Kepler" (p. 5). A comprehensive bibliography contains detailed references to people and subjects discussed in the text.

Constructions for DEMR are presented in *Elements* II,11 and again in VI,30. II,11 involves dividing a line so that the rectangle contained by the whole and the smaller part is equal to the square on the larger part. (The author uses the term "area" throughout his account, although Euclid writes only of the equality of figures.) This result is combined with III,36 and 37 and used in the construction of the regular pentagon in Book IV. Having introduced the theory of proportions in Book V, Euclid is able in VI,def. 3 and VI,30 to present DEMR in terms of ratios. The latter construction is used in Book XIII to inscribe the regular icosahedron and dodecahedron in the sphere.

The author develops the thesis that "III,36 and 37 arose out of the successful attempt (preserved in Euclid IV,10,11) to construct the pentagon and, furthermore, that the concept of DEMR, as presented in II,11, is merely a side product of III,36 and 37 and that II,6 is nothing other than the lemma that was proved as a preliminary to the proofs of III,36 and 37 and II,11" (p. 27). The author assumes "that II,5 and II,6 did not arise out of the development of the theory of incommensurables". He also rejects the suggestion that the material in Book II was a later reworking of Book VI. He considers in detail the question of whether the Euclidean propositions involving "application of areas" (I,44, II,11, VI,28,29) constitute a form of geometrical algebra. His analysis of II,11 and Euclid's *Data* suggests a negative answer, a conclusion based on internal considerations. In reference to the difficult propositions of Book XIII he writes that the purpose of his "descriptions is not to replace Euclid, but rather ... to add a little light and dispel some loneliness" (p. 4).

The author's engagement in his subject is reflected in the wealth of material presented—detailed quotations, references to the literature and mathematical commentary. He remarks on the fascination of reading Euclid: "... the reader is constantly tempted to redo the proofs or rephrase the statements in terms of later developments in the *Elements* or in mathematics and is forever wondering how and why all this came about" (p. 2). In presenting the later history he notes "that it is necessary to compare the approaches of different mathematicians to a very narrow area, such as DEMR, in order really to understand how mathematics has developed" (p. 5).

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