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Laugwitz, Detlef (D-DARM)

★**Bernhard Riemann 1826–1866. (German. German summary)**

Wendepunkte in der Auffassung der Mathematik. [Turning points in the conception of mathematics]

Vita Mathematica, 10.

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This biography contains an introductory chapter on Riemann's life and proceeds to a more detailed study of his contributions to real and complex analysis, differential geometry and applied mathematics. In a concluding chapter the author examines the place of Riemann in the larger development of nineteenth-century and modern mathematics, and includes here sections on Dedekind, Cantor and Hilbert. The book as a whole combines technical explication, piecemeal observation and references to the historical literature in order to create a coherent picture of Riemann and his mathematical milieu.

Although much of the book amplifies on Riemann's well-known achievements in mathematics and physics, Laugwitz brings a particular point of view to his study, one that is more relevant to the predominant mathematical parts than it is to physical subjects. His thesis (indicated in the subtitle) is that Riemann was a central figure in a new conception of mathematics that arose in the nineteenth century. More than anywhere else the origins of this "quiet revolution" were found at Göttingen University and were associated with the names of Gauss, Dirichlet, Dedekind (as a student) and Riemann. An older algorithmic, formal, calculational tradition—represented in the eighteenth century by Euler and Lagrange and in the nineteenth century by such figures as Jacobi and Weierstrass—was succeeded by a more conceptual, intensional mode of doing mathematics. Proofs involving the transformation of formulae or geometrical constructions were replaced by ones derived from conceptual principles and logical thought. Mathematical objects were conceived of as the bearers of conceptual characteristics rather than as analytical expressions or geometrical figures.

Cauchy's and Bolzano's proof of the intermediate value theorem provides a model example of conceptual mathematics. Beginning with the concept of functional continuity these authors were able to establish a property satisfied by a large class of real functions. The result followed not from the manipulation of expressions or the inspection of figures but from a general analysis and unfolding of the concept itself.

Although a detailed examination of the older eighteenth-century tradition would seem to be highly relevant to Laugwitz's thesis, he largely omits consideration of historical writings about the earlier period. In this respect the book is neither systematic nor critical. The foremost nineteenth-century theorist of conceptual mathematical and a philosophical anti-formalist, Gottlob Frege, is not even mentioned. Instead, the narrative concentrates on specific figures in Riemann's biography who may have influenced him. Riemann's understanding of mathematics as thinking about concepts seems to have achieved its distinctive historical form under the influence of the German philospher Johan Friedrich Herbart (1776–1841). Herbart began and ended his distinguished career at the University of Göttingen. Under the name "synechiology" he made the study of the continuous a subject of detailed philosophical investigation. For Riemann, mathematical objects were not simply collections of conceptual properties, but were to be conceived of more fundamentally as elements of a continuous manifold. The continuous and the continuum were for him prior to the discrete; a continuous manifold was a primitive topological conception capable of purely abstract development. In this respect Riemann's thinking ran counter to the prevailing nineteenth-century reductionist trend toward coordinatization and arithmetization. His emphasis on the continuum provided a common intellectual thread connecting his work in such otherwise diverse subjects as complex analysis, differential geometry and field theories in physics.

Riemann's relatively early death (at age 39) and the suggestive, powerful and incomplete character of his writing have made him an almost mythical figure in the history of mathematics. *What would he have done if he'd lived?* His conception of geometry and field-theoretic approach to physics would be realized in Einstein's famous development of the general theory of relativity. Although Dedekind and Cantor had quite different interests and outlooks from Riemann's, Laugwitz suggests that their work was linked to his by a concern for conceptual thinking in mathematics. He also argues that Riemann's conceptual approach was a significant influence on the emergent structural set-theoretic style of modern (Hilbert-Noether-Bourbaki) mathematics. *Craig G. Fraser*