CSHPM Notes brings scholarly work on the history and philosophy of mathematics to the broader mathematics community. Authors are members of the Canadian Society for History and Philosophy of Mathematics (CSHPM). Comments and suggestions are welcome; they may be directed to either of the column's co-editors:

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Les articles de la SCHPM présente des travaux de recherche en histoire et en philosophie des mathématiques à la communauté mathématique élargie. Les auteurs sont membres de la Société canadienne d'histoire et de philosophie des mathématiques (SCHPM). Vos commentaires et suggestions sont le bienvenue; ils peuvent être adressées à l'une des co-rédacteurs:

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"But [the historian's] particular business lies, not with this bare and general similarity, but with the detailed dissimilarity of past and present. He is concerned with the past as past, and with each moment of the past in so far as it is unlike any other moment." – Michael Oakeshott (1933, 106)

Past, Present, and Anachronism in the Historiography of Mathematics

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There is more than one way to view the mathematics of the past. Ivor Grattan-Guinness (2004) identifies a disjunction between heritage (our tracking of a particular concept's journey along the "royal road" from the past to the present) and history (our attempt to explain why a certain mathematical development happened). The "heritage" approach evaluates past mathematics in light of recent theories, looking for similarities that reveal the gradual unveiling of a mathematical concept. Conversely, "history" instinctively looks for differences and discontinuities.

In 1975 a lively debate on the study of ancient mathematics opened with the publication of Sabetai Unguru's "On the need to rewrite the history of Greek mathematics." Unguru addressed what he saw as the anachronism implicit in the then-standard view of Euclid's *Elements Book II* as a form of geometric algebra. The propositions that Euclid proved may be read as geometric versions of algebraic identities, and the central result is the division of a line in the golden section, which may be viewed as equivalent to the solution of a quadratic equation. Nonetheless, Unguru pointed out, there are no algebraic variables or symbols for operations, and the very word "algebra" is of Arabic origin and first appears in Islamic mathematical science over a thousand years after Euclid.

More generally Unguru called for a new and historically sensitive interpretation of the entire corpus of ancient Greek mathematics. Unguru's thesis, which was novel and provocative at the time, was gradually accepted by historical researchers and became something of the received view in the field. It was extended beyond ancient mathematics to medieval and early modern mathematics, and even to the whole range of developments in mathematics since 1700.

The received view, though widely held, is by no means universally endorsed by historians of mathematics. Izabella G. Bashmakova (1993) has written on the history of Diophantine analysis and argues that modern algebraic geometry is required to fully understand the development of this subject over the past two thousand years. In 2014 Victor Blåsjö, a young historian of mathematics, published "A critique of the modern consensus in the historiography of mathematics," in which he advocates for a form of rational history where modern notions play a meaningful role in the interpretation of past mathematics.

An indication of current interest in the historiography of mathematics is evident in an international conference that was held in April 2017 at Caltech: "Anachronism(s) in the History of Mathematics: The Seventh Biennial Bacon Conference." The conference was organized by Niccolò Guicciardini, Professor at the University of Bergamo and recipient of the 2018 Francis Bacon Award in the History and Philosophy of Science and Technology. The subject of the conference was expressed as follows: "Anachronism is often declared the greatest failure, almost a moral sin, a historian can commit. Yet, some have spoken in favor of anachronism, considering it either as an inevitable, or even as a desirable feature of an historical work. The purpose of this two-day international conference is to reflect on the uses and abuses of anachronism in the historical study of the mathematical sciences."

In some preliminary remarks Guicciardini called attention to one of the earliest descriptions of historical anachronism, by Jean Leclerc in his *Ars Critica* of 1697. He also referred to Quentin Skinner's 1969 essay in *History and Theory*, titled "Meaning and Understanding in the History of Ideas," where Skinner observes (p. 9), "We should not credit a writer with a meaning he could not have intended to convey,

since that meaning was not available to him." In his Bacon address Guicciardini discussed the case of Newton's scientific writings and those of his contemporaries, and the narrative tension involved in rendering them comprehensible to a modern reader without compromising their historical character.

In our own contribution to the proceedings of the Caltech meeting we examine some of Leonhard Euler's contributions to mathematics and their later interpretation by mathematicians and historians of mathematics. The two subjects considered are Euler's derivation in the 1740s of the equations of the calculus of variations and his work in the 1750s on divergent series. Certain concepts occupy a fundamental place in the modern subject, but do not appear in the work of either Euler or his contemporaries. In the case of variational calculus there is the concept of the invariance of the variational equations; in the case of infinite series there is the concept of summability. While both concepts are a product of research since the later part of the nineteenth century, modern historical commentators such as Constantin Carathéodory, Herman Goldstine and Morris Kline have discerned the presence of intuitions or embryonic ideas of invariance or summability in Euler's writings. We conclude that claims that Euler grasped invariance, or was a summabilist, are anachronistic. More broadly, we argue that mathematicians and historians may draw on heritage for didactic purposes—say, to teach about convergence. However, for one who takes our modern concepts and methods to be correct, it is easy to slip from a view that Euler ought to have used them to a claim that he did somehow use them. It is then that anachronism reaches the end of its utility: a more historical lens is required to help us see the "past as past" and understand Euler's achievements in their own context.

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