Review

Reviewed Work(s):

Vorlesungen über analytische Mechanik, Berlin 1847/48

by Carl Gustav J. Jacobi and Helmut Pulte

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1814. The main topics he discusses concern geometry. He starts with an analysis of the second part of Georg Simon Klügel's *Mathematisches Wörterbuch,* then outlines the conception of a beginner's textbook of geometry, dealing in particular with space curves and their curvature. And he records his decision to write an article on curvature.

All these reflections give us an insight into how restricted Bolzano's access to the mathematical literature of that time was, for solutions to the problems he examines had already been published. Bolzano did not know about these solutions and derived some results once again. With regard to analysis, Bolzano basically discussed the following three books: *Anfangsgründe der höheren Analysis*, by Johann Gottlieb Friedrich Bohnenberger; Johann Karl Friedrich Hauff's translation of Lazare Carnot's *Réflexions sur la métaphysique du calcul infinitésimal;* and *Leichte und kurze Darstellung der Differential-Rechnung*, by Karl Heribert Ignatius Buzengeiger. Apart from geometrical topics, Bolzano discussed mainly irrational magnitudes, the installation of rigor in analysis, and the concept of function.

The notebook proper is supplemented by a bibliography containing all the references Bolzano makes, a name and a subject index, and Bolzano's own subject index.

KARL-HEINZ SCHLOTE

Carl Gustav J. Jacobi. *Vorlesungen über analytische Mechanik, Berlin 1847/48.* lxviii + 353 pp., illus., figs., index. (Dokumente zur Geschichte der Mathematik, 8.) Deutsche Mathematiker-Vereinigung. Edited by **Helmut Pulte.** Braunschweig/Wiesbaden: Vieweg, 1996. DM 98.

Carl Gustav Jacobi's enduring contribution to mechanics was his development of what is today known as the Hamilton-Jacobi theory. Although the subject is usually presented in modern textbooks as a very formal one, involving techniques for transforming coordinates, it in fact possesses a quite interesting conceptual and theoretical structure. In his dynamical researches of the 1830s, Sir William Hamilton hit upon the idea of taking a certain integral and regarding it as a function of the initial and final coordinate values. He was able to do so because the integration was taken along arcs that were solutions to the dynamical equations. As each point in space is connected by a unique arc of this type, the resulting function is well defined. Implicit in Hamilton's procedure was a powerful concept in the calculus of variations that Oskar Bolza would later (1904) call a "field integral," a notion connected in a fundamental way to the sufficiency theory first developed in the 1870s by Karl Weierstrass.

In 1834 Hamilton reported in a letter to his friend William Whewell that he had "made a revolution in mechanics." Jacobi took what he referred to as Hamilton's "beautiful idea" and developed a revised and improved theory. He emphasized the integration problem and used the theory of partial differential equations to obtain a solution to the dynamical ordinary differential equations in terms of the solution of a corresponding partial differential equation, known today as the Hamilton-Jacobi equation.

Jacobi died in 1851 at age forty-six from an illness related to diabetes. In 1866 Alfred Clebsch published a set of lectures on dynamics that Jacobi had first delivered in Königsberg in the year 1842–1843. In 1843 Jacobi had moved to Berlin and developed a further set of lectures on mechanics there. The book under review is the text of these lectures, now published for the first time, with an introduction and apparatus criticus by Helmut Pulte. The text is transcribed from the remarkably complete notes taken by Jacobi's student Wilhelm Scheibner, who later pursued a mathematical career in Leipzig. (Among the students who attended the Berlin lectures was Bernhard Riemann.)

The 1847–1848 lectures differ from those published by Clebsch in that they contain a detailed presentation of the basic principles of both statics and dynamics. Jacobi begins with a historical overview, then proceeds to an analysis of equilibrium conditions based on the principle of virtual work and Joseph Lagrange's multiplier rule. The dynamical equations are obtained by a conventional application of what is now called "d'Alembert's principle." After an extended discussion of integrals of motion, Jacobi turns to a statement of variational laws, leading up to an exposition of the Hamilton-Jacobi theory. The final lectures deal with the application of this theory to the theory of perturbations in celestial dynamics.

Pulte, who wrote a 1989 book on the history of eighteenth-century theoretical mechanics, provides a full discussion of the historical background to and general character of Jacobi's researches. He shows that the Berlin lectures constituted a break in the development of Jacobi's philosophical thinking about the foundations of mechanics. Before 1847 Jacobi was committed to a largely mathematical point of view inherited from Leonhard Euler and Lagrange, according to which fundamental principles were regarded as a priori postulates of an essentially deductive analytical science. In the Berlin lectures, by contrast, he stressed the empirical, conventional nature of mechanical principles, thereby anticipating, Pulte suggests, the conventionalist doctrines advanced by Henri Poincaré at the end of the century.

As copies of Jacobi's Berlin lectures remained in very limited circulation—Scheibner's manuscript ended up at the Ruhr University in Bochum, and there is a copy as well at the University of Berlin—their influence was largely restricted to the handful of Jacobi's original students and their immediate associates. (The latter group included Scheibner's better-known colleague Carl Neumann in Leipzig.) Pulte's edition will, however, allow a more detailed comparison with the Clebsch edition—historically the important one—and can even serve as something of a commentary to parts of the latter. Jacobi was a major figure in the history of nineteenth-century mathematics, and this edition of his Berlin lectures will help us to clarify both his thought and the mechanical science of the period. Their publication is a central contribution to the history of science. Pulte is to be commended for making the lectures available in an accessible form and for supplementing them with his stimulating commentary and documentary notes.

CRAIG FRASER

June Barrow-Green. *Poincaré and the Three-Body Problem.* (History of Mathematics, 11.) xvi + 272 pp., illus., figs., apps., bibl., index. Providence/London: American Mathematical Society/London Mathematical Society, 1997. $49.

The three-body problem, namely, the determination of the motion of three point masses in mutual gravitational interaction, has attracted the attention of mathematicians of the stature of Isaac Newton, Claude Alexis Clairaut, Leonhard Euler, Joseph Louis Lagrange, Pierre Simon de Laplace, and Carl Gustav Jacobi. In 1885 the journal *Acta Mathematica,* recently founded by the Swedish mathematician Gösta Mittag-Leffler, announced the King Oscar Prize (a gold medal and 2,500 Swedish crowns), to be awarded in January 1889 by Oscar II of Sweden, for his sixtieth birthday. Mathematicians were invited to submit contributions dealing with one of four given topics, the first one being the three-body problem.

Henri Poincaré's memoir entitled *Sur le problème des trois corps et les équations de la dynamique* received the prize at the beginning of 1889, and between July and November 1889 it was printed in the *Acta Mathematica,* with the help of Lars Phragmen, a young assistant editor of the journal. Phragmen found some obscure points in the memoir and wrote several letters to Poincaré. The last one remained unanswered, but on 30 November Poincaré sent the journal a telegram requesting that printing be stopped: he had found a serious error. As issues of the *Acta Mathematica* containing Poincaré's memoir had already been mailed, Mittag-Leffler was forced to ask the subscribers to return them. He saved Poincaré's annotated copy of the first version, however, and, recently discovered in the Mittag-Leffler Institute, it now sheds new light on its past story. The revised version appeared in the *Acta* of December 1890, and Poincaré had to pay the cost of the second printing, an amount greater than the sum he received for the prize!

What Poincaré had first overlooked, and then discovered when correcting his mistake, was that solutions of the three-body problem could behave in an extraordinarily complicated way. This discovery was the first appearance of that paradigm of science known today as chaos theory.

The preceding story is carefully recounted in this monograph by June Barrow-Green. Chapters 1–3 provide a very readable mathematical and historical description of the three-body problem and of Poincaré's early contributions to its solution. Barrow-Green devotes Chapter 4 to King Oscar's competition and the saga of Poincaré's memoir, whose mathematical contents and reception she analyzes in Chapters 5 and 6, respectively. In Chapter 7 Barrow-Green discusses Poincaré's famous monograph *Les méthodes nouvelles de la mécanique céleste,* published between 1892 and 1899 as an expanded version of his prizewinning memoir. The next two chapters present the later contributions of Alexander Liapunov, Tullio Levi-Civita, Paul Painlevé, Karl Sundman, Hugo von Zeipel, George Darwin, Jacques Hadamard, and George Birkhoff, and the last chapter briefly describes some contemporary developments of Poincaré's ideas.

Barrow-Green also includes individual appendixes for each of the following topics: a letter Mittag-Leffler wrote to Sonya Kovaleskaya about the organization of the competition, the official announcement of the prize, the titles of the twelve entries, the report of the Commission, the title and contents pages of both versions of Poincaré's memoir, and a short description of the theorems contained in the first version but not the second. A list of references, an index, and