

MR2874031 01A50 01A55 70-03

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Principles of least action and of least constraint. (English. English summary)

GAMM-Mitt. **34** (2011), no. 2, 164–182.

The article is a lecture that was given to the German Society of Applied Mathematics and Mechanics at its annual meeting in 2010. The 20 figures in the article resemble powerpoint slides that would have accompanied the lecture. The article provides a historical overview of two variational principles, Pierre de Maupertuis' principle of least action (1746) and Carl Gauss' principle of least constraint (1829). Ekkehard Ramm writes (p. 164): "The author not being a historian apologizes for a rather short and partly superficial essay on both principles, based mainly on secondary sources."

Coverage of the principle of least action is more extensive and focuses on the priority dispute that arose from Samuel König's claim (1751) that the idea for Maupertuis' law was anticipated in a letter written by Gottfried Leibniz in 1707. The parties in this dispute were König and Voltaire on the one side, and Maupertuis, Leonard Euler and Frederick the Great on the other. The article gives a good account of the episode, which was of little significance for the history of science. More interesting is Gauss's belief that there was a connection between his constraint principle and Adrien-Marie Legendre's law of least squares, the latter having been applied by Gauss in celestial mechanics in 1809. The article includes some discussion of variational principles in modern mechanics and engineering.

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[References]

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.