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The discovery of the series formula for π by Leibniz, Gregory and Nilakantha.
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The series formula referred to in the title is $\pi/4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$, the special case of the series representation $(*)$ $\arctan x = x - x^3/3 + x^5/5 - \cdots$ that is obtained when $x = 1$. The author reports on the work of Leibniz and James Gregory from the 1670s, and on the work of a 15th-century south Indian mathematician called Nilakantha. Although these researches have already been described in the historical literature, the article provides in one place a detailed and informative account of their different approaches.

Leibniz in the early 1670s derived his famous transmutation rule; in applying it to the area of a sector of a circle he obtained $(*)$. Gregory's researches were completed (apparently) in early 1671 in the course of an investigation in which he used what would later become known as Taylor's theorem to obtain series expansions for several transcendental functions. The author provides a reconstruction of his derivation of $(*)$. He began (in modern notation) with $y = \tan\theta$ and obtained $dy/d\theta = 1 + y^2$. He then set $d\theta = dy/(1 + y^2) = dy(1 - y^2 + y^4 - \cdots)$ and deduced $(*)$ by term-by-term integration.

Nilakantha's results were presented in his *Tantrasangraha*, composed in Sanskrit verse around 1500. An anonymous commentary entitled *Tantrasangraha-vakhya* then appeared, and a century later Jyesthadeva (c. 1500–c. 1610) published a commentary entitled *Yuktibhasa* that contained proofs of the earlier results. The material in the *Tantrasangraha* itself seems to have been the earlier work of Madhava, a mathematician who lived from 1340 to 1425 in Kerala, the southwest coast of India.

Nilakantha derived $(*)$ by obtaining an approximate expression for an arc of the circumference of a circle and then considering the limit. An interesting feature of his work was his introduction of several additional series for $\pi/4$ that converged more rapidly than $1 - \frac{1}{3} + \frac{1}{5} \cdots$. The author provides a reconstruction of how he may have arrived at these results, based on the assumption that he possessed a certain continued fraction representation for the tail series $1/(n+2) - 1/(n+4) + 1/(n+6) - \cdots$.

The three researchers discussed in the article involve different approaches and ideas. The work of Leibniz and Gregory appears to have been independent, and the author is of the opinion that the earlier Indian investigations did not influence 17th-century European mathematicians. (Arabic influences on the Indians seem however to have been probable.) He also notes that the Japanese mathematician Takebe Kenko (1664–1739) apparently independently derived an infinite series for π in 1722. He observes that "the independent discovery of the infinite series by different persons living in different environments and cultures gives us insight into the character of mathematics as a universal discipline". (The appearance of universality is of course accentuated by the author's decision throughout to render in standard modern notation the very different styles of the original mathematical work.) *Craig G. Fraser*