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Timmermans, Benoît (B-ULB-FNR)

★**La méthode cartésienne face aux questions numériques. (French) [[The Cartesian method in the face of numerical questions]]**

Descartes et ses mathématiques, 113–134, *Hist. Philos. Sci.* (*Paris*), 27, *Classiques Garnier*, *Paris*, 2022.

Results about odd perfect numbers were discussed by René Descartes in letters from the late 1630s to Marin Mersenne and Frenicle de Bessy. He hypothesized that odd perfect numbers will be of the form k^2p, where p is prime and k is the product of distinct prime numbers. He proposed $k^2p = 3^2 \cdot 7^2 \cdot 11^2 \cdot 13^2 \cdot 22021$ as a possible perfect number. Unfortunately, 22021 is equal to $61 \cdot 19^2$, but this fact (which Descartes seemed to have become aware of at some point) did not deter him. He devised an algorithm to generate candidates for odd perfect numbers which involved the use of superparticular ratios (ratios of the form $n : (n+1)$). This is described in some detail by Benoît Timmermans using modern notation and arithmetic functions.

Descartes' interest in number theory wavered but was more pronounced at the end of the 1630s. It is not indicated in this article what, if any, public dissemination of his work on perfect numbers occurred, and indeed to what degree it was of any historical consequence. (Descartes' writings in question were only published at the end of the nineteenth century.) This situation may have been in part a reflection of the limited character of the research subject. Descartes' letters do highlight aspects of his way of thinking and working that are relevant to his intellectual outlook and to his writings that were published. Here Timmermans situates his discussion in relation to the considerable historical and philosophical literature on Descartes of the past several decades.

Descartes focused primarily on proportions and relations between numbers rather than on the numbers themselves. He displayed a certain pragmatism in his approach to problem solving and was willing to begin with a method he knew to be flawed but which led in practice to results. He also recognized an affinity between arithmetic ratios and the musical scale. Here the notion of a superparticular ratio which arose in Descartes' treatment of possible odd perfect numbers played a role.

Reference in this paper is made to a fairly recent article on perfect numbers that is related to Descartes' work on odd perfect numbers and provides a modern perspective on it [W. D. Banks et al., in *Anatomy of integers*, 167–173, CRM Proc. Lecture Notes, 46, Amer. Math. Soc., Providence, RI, 2008; MR2437973]. *Craig G. Fraser*